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Getafe, Junio 2017

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Chapters

Country default in a monetary union

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Abstract

We develop a simple model of borrowing and lending within the monetary union. We characterize the default decision of the borrowing country and explore the impact that the monetary union has on the amount of borrowing, the rate of interest and the default probability. The key assumptions of the modelling strategy are that in the monetary union, the lender is risk averse with monopoly power rather than risk neutral with perfect competition. We find that the borrowing member country of the monetary union borrows more at cheaper cost vis-à-vis a standalone borrowing country. Further, we find that forming a monetary union with high initial income disparity between the member countries leads to more and cheaper borrowing and higher default probabilities.

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1 Introduction

The third stage of the European Economic Monetary Union (EMU) in 1999 led to the adoption of the Euro as a common currency among member countries and to their surrendering of national monetary policy in favour of the European Central Bank. The adoption of the Euro triggered a dramatic increase in external net borrowing among peripheral countries such as Greece, Italy, Portugal and Spain. The main lenders to these countries were France and Germany which together held around 45% of the Portuguese and the Spanish debt during the period 2001-2013. More specifically, around 77% of the French debt holdings were issued by countries of the European Union and 67% were issued by Eurozone countries. Likewise, 77% of the German debt holdings were issued by the European Union countries and 66% were issued by the Eurozone. The financial crisis of 2008, along with the above-mentioned high exposure of France and Germany to the debt of peripheral countries, increased the risk of default among the borrowing countries and led to the Greek debt crisis of 2010.

In this paper we present a model of borrowing and lending to study how a monetary union shapes the amount of borrowing, its price and the default probability. In particular, we investigate how the debt market responds to changes in both the costs and benefits associated with a monetary union. Our framework is also useful to shed light on how high initial income disparity between countries in a monetary union affects borrowing, lending and default. This is a question that has gained recent attention in light of the debt crisis of peripheral countries. Our model builds on existing literature on debt default (Eaton and Gersovitz, 1981 and Arellano, 2008). We extend this literature by introducing a monetary union which we model as a technology that changes the income processes of the member countries. We describe a simple economy with two countries that receive exogenous income streams every period. Debt contracts are not enforceable and hence the borrowing country can choose to default on its debt. In case of default, the union breaks and both countries revert to autarky without any future interaction in the international credit market. A second main departure from the previous literature is that we model the lender as being risk averse with monopoly power, instead of a continuum of risk neutral lenders operating in a perfectly competitive market. The lender is also subjected to exogenous income shocks, which depend on whether it belongs to the monetary union or not. We model the benefit of a monetary union by assuming that the growth rate of income is higher relative to autarky. We model the cost of a monetary union by assuming higher income volatility than in autarky. Empirical evidence supporting this modelling strategy is provided by several papers.

We find that higher amount of borrowing at lower yield formed the equilibrium in the monetary union vis-à-vis standalone. Any two standalone countries engage in borrowing and lending only when the default cost is quite high. The costs and benefits associated with the monetary union add another level of trade-off with respect to the default decision. This not only increases the debt supply by the borrowing country in the monetary union but also increases the amount borrowed in equilibrium. The model yields additional result that higher income disparity between member countries leads to more and cheaper borrowing in the monetary union. The equilibrium outcomes, as were observed in the EMU debt market are the result of interplay between the market structure and agents' characteristics. The results when translated into policy recommendations call for either expanding the joining criterion to the real variables such as income disparity or developing tools to counter higher debt issuance when cross country income disparity is higher.[1]

2 Some data on borrowing-lending within the EMU

In this section we present the empirical evidence on borrowing and lending within the EMU. We focus our attention on the debt securities issued by Greece, Italy, Portugal and Spain. Specifically, we look at the net lending by France and Germany to the above mentioned countries.

We use data from the International Monetary Fund's Coordinated Portfolio

[1]This is line with the optimal currency area literature, seminal work by Robert Mundell (1961), where convergence of the real variables is crucial for the overall stability of a common currency area.

Investment Survey from 2001 till 2013 (CPIS, 2013).[2] This is an annual survey offering data on portfolio investments by the residents of a reporting country in the debt securities, short- and long-term instruments valued at market prices, of the issuing country. The CPIS collects data either on a security-by-security basis or on an aggregate basis.[3] It uses a "from-whom-to-whom" approach and compiles information from either the end-investors or from custodians, or from a combination of the two. In an end-investor survey, the security owner reports directly, while in a custodian survey, financial institutions that hold securities report on behalf of the end-investors.[4] The holders of the debt securities are either general government or financial institutions or non-financial corporations. General government consists of the central, state and local governments, social security funds, non-profit institutions and unincorporated enterprises that are controlled by the government units.[5]

We focus on short- and long-term debt securities. Short-term debt securities consist of money market instruments that yield the holder a fixed payment on a particular date, and that matures in less than a year. These include treasury bills and notes, commercial and financial paper, and bankers acceptances, negotiable certificates of deposit, short-term notes issued under note issuance facilities or revolving underwriting facilities and promissory notes, debt securities that have been sold under repurchase agreements and debt securities that have been "lent" under a securities lending arrangement. Long term debt securities consist of

[2]It is worth noting that both the CPIS and the QEDS (Quarterly External Debt Survey) are the databases on the private and public external debt. There is a discrepancy between the aggregate values of the total investment in the debt securities reported by each of them. While the CPIS reports not only the debt securities holding and the issuing countries but also the investors' profile in the reporting country, QEDS has only aggregate information with respect to the issuing country of the debt.

[3]A security is defined as a tradable instrument and is identified by the International securities identification number. We exclude equity securities for our analysis. Equity covers all instruments which are shares and stocks, participation documents, depository receipts, shares in mutual funds and investment trusts, securities that have been sold under repurchase agreements, lent under a securities lending arrangement etc. Financial derivatives and related non-resident enterprises (an enterprise group which has an equity interest of 10% or more or where a non-resident has more than 10% or more holdings in your group) are excluded from the survey.

[4]End-investors includes institutional investors, such as banks, security dealers, mutual funds, and pension and insurance funds. Custodians are also financial institutions but they manage securities on behalf of domestic residents.

[5]Governments are majorly the issuers of debt instruments rather than the buyers.

bonds, debentures, and notes with maturity longer than a year. These include "straight" coupon bonds, non-participating preferred stocks or shares, convertible bonds and bonds with optional maturity dates, negotiable certificates of deposit, dual currency bonds, zero-coupon bonds and other deep discount bonds, floating rate bonds (FRNs), indexed bonds (IBs), asset-backed securities (ABSs), euro medium-term notes, Schuldscheine notes, debentures, bearer depository receipts denoting ownership of debt securities issued by non-residents, debt securities that have been sold under repurchase agreements and debt securities that have been "lent" under a securities lending arrangement.[6]

We use information on holdings and issuances of debt securities to calculate *ownership percentage* which is defined as the share of a lender in the total debt issuance of the borrower,

$$\text{Ownership percentage} = \frac{\text{debt securities issued by the borrower and owned by a lender}}{\text{total debt securities issued by the borrower}}$$

We also construct *exposure percentage*, which offers a measure of a lender's exposure to the debt securities of a given issuer in its portfolio,

$$\text{Exposure percentage} = \frac{\text{securities issued by a borrower and owned by the lender}}{\text{total debt securities owned by the lender}}$$

We now report net lending by France and Germany to Greece, Italy, Portugal and Spain from 2001 till 2013. We show net lending for two sub-periods: from 2001 until 2009 and from 2009 until 2013. The first sub-period corresponds to post-EMU formation and the second sub-period corresponds to the ongoing EMU debt crisis. Figures 1 to 4 illustrate an increase of more than 400% in net lending by France to Greece, Italy, Portugal and Spain during the first sub-period.[7] It grew from USD 10 bn in 2001 to USD 80 bn in 2009 for Greece; from USD 36 bn in 2001 to USD 207 bn in 2009 for Italy; from USD 8 bn in 2001 to USD 60 bn in 2009 for Portugal and from USD 21 bn in 2001 to USD 175 bn in 2009 for Spain. The net lending by France stalled at the onset of the EMU debt crisis. It

[6] FRNs such as perpetual-rate notes, variable-rate notes, structured FRN, reverse FRN, collared FRN, step-up recovery FRN, and range/corridor/accrual notes. IBs such as property index certificates. ABSs such as collateralized mortgage obligations and participation certificates.

[7] The net lending is valued at the market prices.

fell by 93% for Greece; by 72% for Portugal and by 8% for Spain from 2009 till 2013. Even though, Italy's debt market recovered immediately after the drop in 2009, the net lending grew at a meagre 9%.

Figures 5 to 8 report net lending by Germany to the aforementioned borrowing countries. Figure 5 plots net lending to Greece and it increased from USD 14 bn in 2001 to USD 38 bn in 2009, an increase of 156%. Post-debt crisis the net lending plummeted to USD 7 bn in 2013. While, we observe in figure 6 that net lending to Italy grew by 250% from 2001 until 2013, it dropped in 2003, 2005 and 2011. Figure 7 and 8 document the increase in net lending to Portugal and Spain by more than 450 % during the first sub-period. Up until 2002, the net lending to Spain was negligible; however, by 2007 it reached the peak at USD 178 bn. During the second sub-period net lending stalled for Portugal and Spain.

We present the evidence in support of two central assumptions of our theoretical model: risk averse lenders with monopoly power over the debt securities market of the borrower. Figures 9 and 10 show the ownership percentage of France and Germany in the debt issuance of Greece, Italy, Portugal and Spain. Germany remained one of the top holders of the Spanish debt securities with ownership percentage around 20% from 2001 till 2013. During the same period, Portuguese debt was held mostly by France with an ownership percentage around 30%.

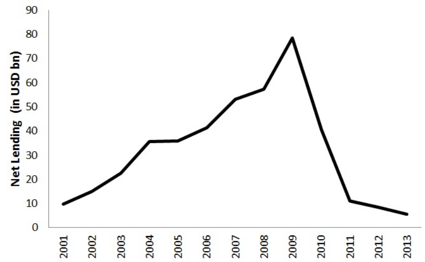


Figure 1: France to Greece

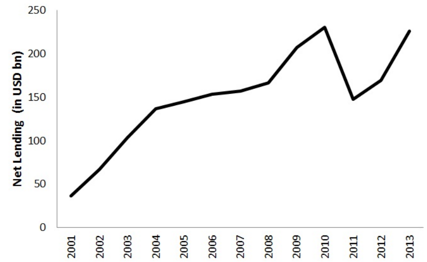


Figure 2: France to Italy

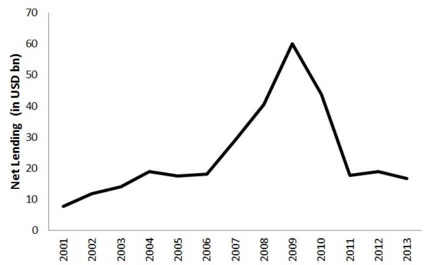


Figure 3: France to Portugal

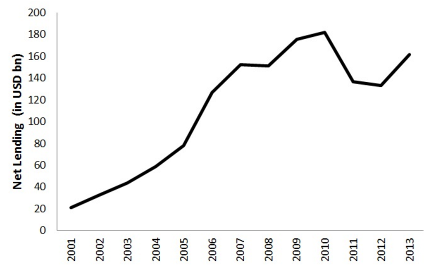


Figure 4: France to Spain

Source: Coordinated Portfolio Investment Survey (IMF).

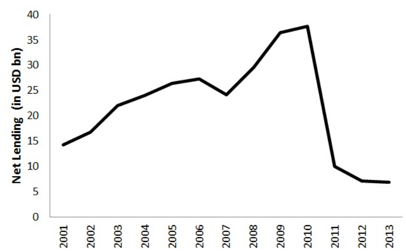


Figure 5: Germany to Greece

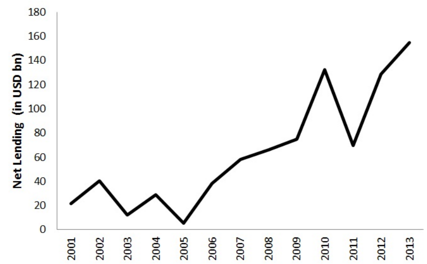


Figure 6: Germany to Italy

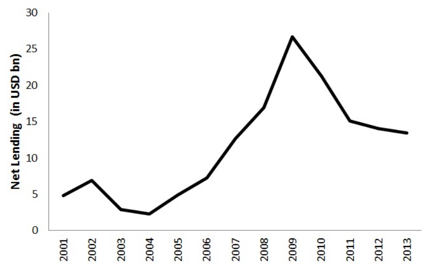


Figure 7: Germany to Portugal

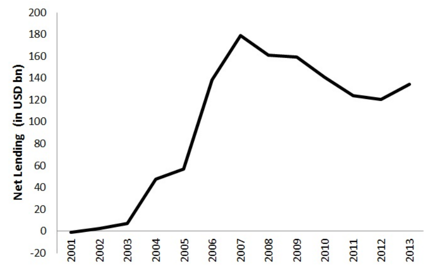


Figure 8: Germany to Spain

Source: Coordinated Portfolio Investment Survey (IMF).

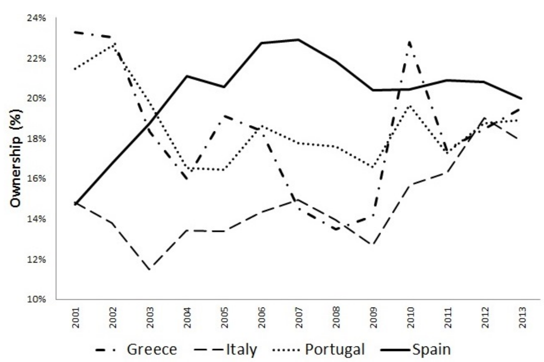


Figure 9: Germany

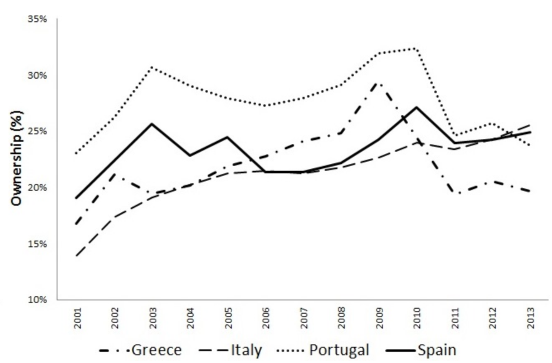


Figure 10: France

Source: Coordinated Portfolio Investment Survey (IMF).

The right axis of figures 11 and 12 plots the Portuguese and Spanish total debt securities and the ownership percentage for major lenders in the debt securities markets on the left axis.[8] We observe that France and Germany held 40-50%

[8]The figure plots the total debt securities values in logs.

of the Portuguese and Spanish debt securities consistently from 2001-2013. The holdings of Spanish debt by the Euro countries dwarf the holdings by non-Euro countries (US, UK, Japan, Norway and rest of the world). We measure the Portuguese and Spanish debt market concentration by calculating the Herfindahl-Hirschman index (HHI).[9] Table 1 reports the HHI for all lenders and for the Euro zone lenders, which participated in the Portuguese and Spanish debt market. The HHI shows that debt markets have been moderately concentrated within the Eurozone countries.

Figures 13 and 14 plot the exposure percentages for France and Germany. On the left axis we document six of the debt issuing countries for which the exposure percentage was more than 5% from 2001 till 2013. The debt portfolio of France and Germany consist of securities issued by Italy, Netherlands, Spain, the United Kingdom and the United States. France and Germany also invested in each other's debt securities. On the right axis we plot the total debt held by France and Germany with exposure percentage greater than 1%.[10] France and Germany had huge exposure to the debt securities of few countries. They held more than 75% of the debt securities issued by member countries of the European Union and more than 65% of the debt securities issued by the Eurozone countries. Further, more than 90% of the debt securities was issued by the European Union countries, Japan, United States and United Kingdom alone.

[9]HHI is a measure of the size of a lender in relation to the overall market. It is the sum of square of the market share of all the players. It ranges from close to zero to 10,000. A higher number indicates less competitive market.

[10]In the case of Germany, debt securities of Austria, Belgium, Denmark, Finland, France, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, United Kingdom, United States, Japan, Cayman Islands, Jersey and International Organizations.

France held securities of Austria, Belgium, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Netherlands Antilles, Portugal, Spain, Sweden, United Kingdom, United States, Japan, Cayman Islands, International Organizations.

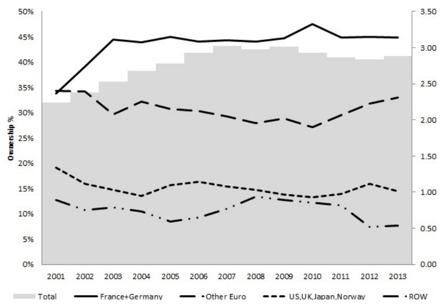


Figure 11: Spain

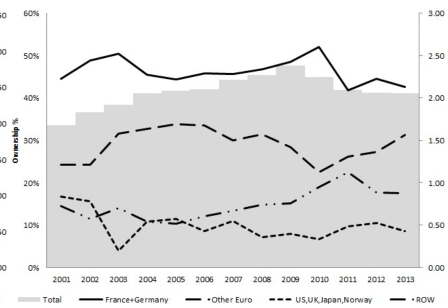


Figure 12: Portugal

Source: Coordinated Portfolio Investment Survey (IMF).

Years	Spain		Portugal	
	All lenders	Euro Zone	All lenders	Euro Zone
2001	947	1643	1283	2017
2002	1114	1778	1472	2182
2003	1268	2003	1577	1931
2004	1274	1900	1429	1926
2005	1347	1993	1386	1865
2006	1280	1973	1372	1834
2007	1266	2026	1359	1943
2008	1266	2057	1434	1894
2009	1276	2022	1540	2041
2010	1382	2159	1674	2096
2011	1269	1961	1259	1726
2012	1290	1917	1388	1768
2013	1317	1918	1327	1613

Table 1: Herfindahl-Hirschman index
Source: Coordinated Portfolio Investment Survey (IMF).

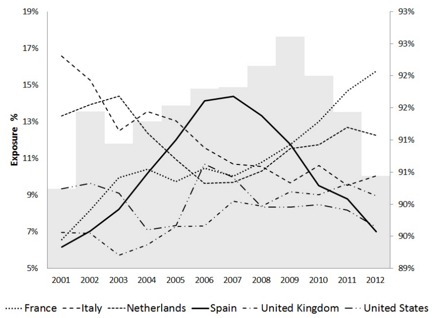


Figure 13: Germany

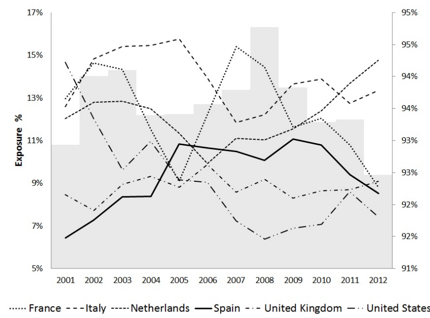


Figure 14: France

Source: Coordinated Portfolio Investment Survey (IMF).

3 The Environment

In the following section we will consider an economy with two countries, $i \in \{l, b\}$, which are inhabited by risk averse households of unit mass each. The households are identical within each country. Time t is discrete and infinite with lending and borrowing taking place in period, $t = 1$. There are two alternate regimes in our analysis, $j \in \{m, k\}$, where m represents the monetary union and k the autarky. The monetary union can be thought of as a technology that increases a country's potential income (higher trend) at the cost of giving up the ability to smooth income shocks (higher volatility).

Households The preferences of the representative household in country i are

$$E_1 \sum_{t=1}^{\infty} \beta_i^{t-1} u(c_{i,t}^j), \tag{1}$$

where $\beta_i \in (0, 1)$ is the household discount factor and $c_{i,t}^j$ is the consumption of the household in country i when it is in regime j in period t. The utility function is strictly increasing and concave, i.e. $u'(.) > 0$ and $u''(.) < 0$ and satisfies the standard Inada conditions. Households receive a stochastic endowment, $y_{i,t}^j$, every period and a lump sum transfer of goods from its government, a_t.

The income process When country i is in regime j at any period t, income (endowment) follows the process,

$$log(y^j_{i,t+1}) = \alpha^j_i.(t+1) + \rho_i log(y^j_{i,t}) + \epsilon^j_{i,t+1}, \tag{2}$$

where, the trend parameter α^j_i is positive, the persistence parameter ρ_i is within the unit circle and the income shocks $\epsilon^j_{i,t+1}$ are i.i.d and normally distributed $N(0, \varsigma^{j2}_i)$. ς^j_i is the standard deviation of the income shocks.

The income trend is higher for the member countries in the monetary union, which is attained through channels such as reduction in transaction costs, enhanced trade, financial deepening, seigniorage gains etc. (Papaioannou and Portes, 2008 and Frankel and Rose, 2002). At the same time, a higher income volatility is observed for the member countries on account of losing the monetary policy discretion (Luque et al., 2014 and OECD, 1999) and use of a single monetary policy when faced with asymmetric shocks across the member countries (Werning and Farhi, 2012). Thus,

$$\alpha^m_i > \alpha^k_i \quad \text{and} \quad \varsigma^m_i > \varsigma^k_i \quad \text{for} \quad i \in \{l, b\} \tag{3}$$

Governments The government of each country is benevolent and maximizes the utility of the representative household. In period $t = 1$ both countries are in the monetary union. They can smooth their consumption by trading a non-contingent bond which pays a time- and state-invariant return. We assume that $\beta_b < \beta_l$, so that country b is the net borrower and country l is the net lender. Also, we assume that the two countries have some initial asset/liability, a_1, which has to be repaid by the borrowing country in period $t = 1$.

Borrowing and Lending In period $t = 1$, the borrowing country borrows an amount a_2 at a given discount q_2, from the lending country.[11] The borrowing country lacks commitment to repay its debt obligation in period t=2. The default is an absorbing state, following which the monetary union breaks. The borrowing

[11]Note here, q_2 represents the bond price or the discount at which the bond is sold and $\frac{1}{q_2}$ represents the interest rate for the same bond.

country is immediately penalized with γ fraction of its income realization in period $t = 2$. If however the debt is repaid, the monetary union continues to exist.

In Section 2 we provided the evidence that in the EMU only a few lending countries held most of the debt securities issued by the borrowing countries (CPIS, 2013). We follow this empirical finding and assume, for simplicity, that the single lending country is the sole source of borrowing and it has the monopoly power over the lending market instead of a continuum of lenders in a competitive market. In period $t = 1$, the lender chooses the bond price q_2 for a given debt. It remains in the monetary union if the borrowing country repays the debtin period $t = 2$.

3.1 Evidence on the "monetary union" technology

The "monetary union" is modelled as a technology, which allows member countries attain a higher long run growth in income at the expense of a higher income volatility. This reduced form definition of the monetary union captures parsimoniously the benefits and the costs of adopting a common currency.

In this section we assess the effects that the launch of the euro has on its member countries. Generally speaking, a monetary union benefits its members directly and indirectly through trade and financial market deepening. A common currency lowers the transaction costs, reduces price uncertainty and enhances the price transparency. At the same time adopting a common currency not only takes discretionary monetary policy away from each member country but also leave them facing asymmetric shocks with a single union-wide monetary policy.[12]

Frankel and Rose (2002) use a two-stage approach to quantify the effects of monetary unions on income through trade. They use cross-sectional data from over 180 countries for the period from 1970 until 1995. They provide econometric evidence that a monetary union triples trade. Further, an increase in total trade-to-GDP ratio by one percent raises income per capita by at least one-third of a percent.[13]

There are several papers estimating particularly the effect of the euro on trade.

[12]For a textbook treatment refer to De Grauwe (2012).

[13]The first estimate on the effect of currency unions on trade is larger if the currency union partnership has linguistic, historical, political and geographical links.

In a survey of the literature by Baldwin (2006), he concludes that the euro led to an increase in trade by five to ten percent within the euro area. Micco et al. (2003) use a panel data for 15 Euro countries from 1992 to 2002 and apply the difference-in-differences estimate approach. They find that the effect of the euro on trade between member countries is between 4-10%. The monetary union also enhances trade with non-Euro countries. Flam and Nordström (2006) estimate the increase in trade by 15% within the Euro area and by 8% with the other EU countries from 1989-2002 until 1998-2002.

Another channel which influences the output growth is through internationalization of the Euro. It reaps benefits to the monetary union in the form of seigniorage and expansion of the financial markets. Papaioannou and Portes (2008) discuss the positive effect of Euro adoption on deepening and integration of the financial markets. The financial market development affects the economic performance through channels such as risk sharing, lowering cost of capital, fast reallocation of investments etc.

We use data from the Organisation for Economic Co-operation and Development (OECD) database to calculate three-year averages of the real GDP's annual growth rates for France, Germany, Greece, Italy, Portugal and Spain, before and after joining the Euro.[14] We observe that after joining the Euro the annual growth rate increased from 3.8% to 4.5% for Greece; from 1.5% to 2.3% for Italy; from 3.6% to 4.5% for Spain; from 2.4% to 3.1% for France and from 1.5% to 2.2% for Germany but it declined from 4.2% to 3.2% for Portugal.

Regarding the income volatility, Luque et al. (2014) report average volatilities of GDP per capita before and after the adoption of the euro for the periods, 1986-1998 and 1999-2011. They document that average volatilities increased after 1999 for almost all member countries. It increased from 6.67% to 14.94% for Greece; from 10.63% to 13.5% for Italy; from 19.38% to 23% for Portugal; from 9.54% to 18.91% for Spain and from 4.6% to 7.8% for France. However, it declined from 5.61% to 4.02% for Germany.

One of the explanations behind this increase in income volatility is that some

[14] France, Germany, Italy, Portugal and Spain joined the Eurozone in 1999 while Greece joined in 2001. For details on data refer to FRED or OECD (*http://dx.doi.org/10.1787/data-00052-en*).

member countries not only had high inflation variance historically but also were structurally different from other member countries. After adopting a common currency it is impossible to adjust the exchange rate in order to accommodate the inflation variation. Thus a union-wide single monetary policy would lead to higher output variability. Another mechanism leading to the increase in output volatility is that asymmetric shocks across member countries cannot be targeted by a single monetary policy (Lane, 2012; Shambaugh et al., 2012 and Werning and Farhi, 2012).[15]

3.2 Characterization of borrowing and lending

We solve the model by using the method of backward induction. We consider three distinct stages. In the first stage, the borrower takes the decision of default or repayment, given the amount of debt, a_2, and the price, q_2, paid for this amount. In the second stage, the borrower anticipates the default probability, $\delta(a_2, q_2)$, and chooses the amount of debt, $a_2(q_2)$, given the price, q_2. Finally, in the last stage, the lender determines the price, q_2, to offer, with the anticipation of both the default probability, $\delta(q_2)$, and the debt amount, $a_2(q_2)$.

Let us denote the value function of the borrowing country in period $t = 1$ as $V_{b,1}$. The maximization problem of the borrowing country is,

$$V_{b,1} = \underset{\{a_2\}}{Max} \left\{ u(y^m_{b,1} - a_1 + a_2 q_2) + \beta_b E_1 \left[\underset{d_2 \in \{0,1\}}{Max} \, \mathcal{I}_{\{d_2=1\}} V^k_{b,2} + \mathcal{I}_{\{d_2=0\}} V^m_{b,2} \right] \right\}, \tag{4}$$

where $\mathcal{I}$ is the indicator function, d_2 represents borrowing country's decision to default ($d_2 = 1$) or repay ($d_2 = 0$) and it depends on the income realization in period $t = 2$. $V^k_{b,2}$ and $V^m_{b,2}$ are the value functions of the borrower in period $t = 2$ associated with the default and the repayment decision.

$$V^m_{b,2} = u(y^m_{b,2} - a_2) + \beta_b E_2 \left[V^m_{b,3} \right] \text{ and } V^k_{b,2} = u((1-\gamma) y^m_{b,2}) + \beta_b E_2 \left[V^k_{b,3} \right],$$

[15] For a discussion on the effect of the Euro through a political economy channel see Fratzscher and Stracca (2009).

where, $V^k_{b,3}$ and $V^m_{b,3}$ are the continuation values of the borrower in period $t = 3$ in the two monetary regimes and are discussed in detail later.

The value function of the lending country in period $t = 1$ is denoted by $V_{l,1}$. The maximization problem of the lending country is

$$V_{l,1} = \underset{\{q_2\}}{Max} \left\{ u(y^m_{l,1} + a_1 - a_2 q_2) + \beta_l E_1 \left[\mathcal{I}_{\{d_2=1\}} V^k_{l,2} + \mathcal{I}_{\{d_2=0\}} V^m_{l,2} \right] \right\}, \tag{5}$$

where $V^k_{l,2}$ and $V^m_{l,2}$ are the value functions of the lending country in period $t = 2$ corresponding to the default and the repayment decision of the borrowing country.

$$V^m_{l,2} = u(y^m_{l,2} + a_2) + \beta_l E_2 \left[V^m_{l,3}\right] \text{ and } V^k_{l,2} = u(y^m_{l,2}) + \beta_l E_2 \left[V^k_{l,3}\right],$$

where, $V^k_{l,3}$ and $V^m_{l,3}$ are the continuation values of the lender in period $t = 3$.

Continuation Value For all the periods $t > 2$, the lending and the borrowing country receive utility from consuming the income. The income realizations depend on the regimes that they are in, i.e. whether the borrowing country defaulted and remained forever as standalone or repaid the debt and remained forever in the monetary union. For a given i and j, let $E_2 \left[V^j_{i,3}\right] = \widetilde{V}^j_i$, be the expectation of continuation value of period $t = 3$ when in period $t = 2$.

Under the case of CRRA preferences, for a given i and j, the continuation value is[16]

$$\widetilde{V} = \widetilde{V} \left(\beta, \sigma, \rho, \varsigma, \alpha, y_2\right).$$

Specifically, for the logarithmic preferences the continuation value is,

$$\widetilde{V} = \left[\frac{\rho log(y_2)}{1-\beta\rho} + \frac{\alpha}{1-\beta\rho}\frac{3-2\beta}{(1-\beta)^2}\right].$$

[16]The super and sub-scripts, i and j are suppressed for more clarity. The functional form of $\widetilde{V}\left(\beta, \sigma, \rho, \varsigma, \alpha, y_2\right)$ is described in the appendix.

Result 1 *When the output trend is strictly positive, the continuation value* $\widetilde{V}_i^j$ *exist iff* $\sigma_i \geq \widetilde{\sigma}_i$*, where* $0 \leq \widetilde{\sigma}_i < 1$.

Proof in Appendix.

Decision of default/ repayment at period $t = 2$ We consider the default decision of the borrowing country for a given borrowed amount, a_2 and price, q_2. The borrowing country chooses to repay the loan in period $t = 2$, depending on whether the value of repayment is greater than the value of default in period $t = 2$, i.e.

$$V_{b,2}^m \geq V_{b,2}^k \qquad \Leftrightarrow \qquad y_{b,2}^m \geq \widehat{y}, \tag{6}$$

where $\widetilde{\Omega}_b = \widetilde{V}_b^k - \widetilde{V}_b^m$ and $\widehat{y} = \widehat{y}(\beta_b, \gamma, \widetilde{\Omega}_b, a_2)$.

The default decision of the borrowing country depends on the income realization in period $t = 2$. If the realized income is higher than the threshold, $\widehat{y}$, the borrowing country will repay the loan. The income threshold is increasing in the level of borrowed amount. Alternatively, we can also use the above equation to define a threshold, $\widehat{a}_2 = \widehat{a}_2(y_{b,2}^m)$, at which the borrower is indifferent between its decision to either default or repay the debt in period $t = 2$. We define the repayment set as a set of debts where $a_2 \leq \widehat{a}_2$ and the default set as a set of debts where $a_2 \geq \widehat{a}_2$.

For the CRRA preferences, the debt level for which the borrower is indifferent between default and repayment is

$$\widehat{a}_2 = y_{b,2}^m - \left[((1-\gamma)y_{b,2}^m)^{1-\sigma_b} + \beta_b \widetilde{\Omega}_b\right]^{\frac{1}{1-\sigma_b}}.$$

For the logarithmic preferences,

$$\widehat{a}_2 = y_{b,2}^m \Big(1 - (1-\gamma)\exp(\beta_b \widetilde{\Omega}_b)\Big).$$

In the proposition below we will see the effect of changes in the parameters and the income realizations on the debt threshold $\widehat{a}_2$. We assume that in the benchmark economy the borrowing country's representative household has logarithmic preferences.

Proposition 1 *For a borrowing country with logarithmic preferences, everything else remaining same,*

i) a higher difference in the output growth, $\alpha_b^m - \alpha_b^k$, expands the repayment set (or contracts the default set) for all levels of income,

ii) a higher standalone income realization, $y_{b,2}^k$, leads to contraction of the repayment set (or expansion of the default set),

iii) a higher monetary union income realization, $y_{b,2}^m$, leads to expansion of the repayment set (or contraction of the default set) if $\rho_b > \frac{1}{2}$.

Proof in Appendix.

The above results remain robust if the borrowing country has CRRA preferences. We find that for a coefficient of relative risk aversion greater than one, a higher amount of debt will be repaid in the monetary union if and only if the income volatility difference, $\varsigma_b^m - \varsigma_b^k$, is lower. The results are intuitive as the relative benefits (or the relative costs) associated with the monetary union vis-à-vis standalone country goes up (or down), the value of repayment becomes higher relative to the value of default. Thus, the debt threshold, for which the two values are same, goes up as well and the repayment region expands. Further, we find that an increase in the coefficient of relative risk aversion decreases the debt threshold $\widehat{a}_2$. A higher risk aversion indicates a lower utility due to uncertainty, ceteris paribus. The value of default goes up for a higher risk aversion and thus the repayment region shrinks. The evidence in support of these assertions are provided in the appendix.[17]

We also explore the question whether the borrowing country has a larger repayment region when it is in the monetary union vis-à-vis standalone country. We assume that the default choice of the borrowing country, which is not a

[17]For any numerical analysis we consider the parameter space where, $\alpha_b^m \in [0.015, 0.02]$, $\alpha_b^k \in [0.0, 0.015]$, $\varsigma_b^m \in [0.0035, 0.007]$, $\varsigma_b^k \in [0, 0.0035]$, $\sigma \in \{1, 1.05, 1.15, 2\}$ and the support for incomes are around 0.85, i.e. $y_{b,2}^m \in [0.7, 1]$ and $y_{b,2}^k \in [0.7, 1]$.

member of the monetary union, results in immediate output cost. The income of the standalone country follows the same process irrespective of its default decision and thus the continuation values are same.

Proposition 2 *In our benchmark economy the default set is smaller under the monetary union, ceteris paribus.*

Proof in Appendix.

Corollory *For a given debt level, a_2, the 'indifference' income threshold is smaller in the monetary union when the output penalty is small.*

Proof in Appendix.

A higher income trend, which is made possible by being in the monetary union, raises the value of repayment of the member borrowing country. This is not true in the case of a standalone country and the default region is bigger.

Default probability at period t **=*1*** The borrowing and the lending countries anticipate default in period $t = 1$. The borrowing country defaults if and only if its income realization in period $t = 2$ is lower than the income cutoff, i.e. $y^m_{b,2} \leq \widehat{y}$. Thus, the default probability is $\delta_2(a_2) = \Pr\left[y^m_{b,2} \leq \widehat{y}\right]$.

Result 2: *The default probability,*

$$\delta_2(a_2) = \frac{1}{2}\left[1 + erf\left(\frac{log\left(\frac{\widehat{y}}{(y^m_{b,1})^{\rho_b}\exp(2\alpha^m_b)}\right)}{\sqrt{2}\varsigma^m_b}\right)\right], \tag{7}$$

is non-negative and goes to zero (or one) for a very low (or high) income threshold $\widehat{y}$. The default probability is increasing in the borrowed amount, a_2.

Proof in Appendix.

In the above formula, $erf(.)$ represents the error function. It is a special function, which is increasing in its argument and lies between zero and one in its domain of positive real numbers.

Proposition 3: *In our benchmark economy, for a given level of borrowing,*

a_2, the default probability is lower in the monetary union vis-à-vis standalone countries when the output penalty, γ, is small.

Proof: Using the corollary of Proposition 2, for any given level of debt, a_2, it is immediate that the default probability in monetary union is lower whenever the output penalty is small. A detailed proof is available in the appendix.

Choice of the level of debt in period $t =$***1*** The borrowing country chooses the amount of debt, given the bond price q_2 to maximize its lifetime utility given in Eq (4).

The debt supply by the borrowing country depends on two debt thresholds, $\overline{a}_2$ and $\underline{a}_2$. The first threshold, $\overline{a}_2$, corresponds to the case where even if the borrowing country receives the highest possible income shock, $\overline{y}^m_{b,2}$ it will default. Similarly, the second threshold, $\underline{a}_2$, corresponds to the case where even after receiving the lowest possible income shock $\underline{y}^m_{b,2}$ it will not default. Formally the debt thresholds are expressed as,

$$\overline{a}_2 = \left\{a_2 \left| V^m_{b,2}(\overline{y}^m_{b,2}) < V^k_{b,2}(\overline{y}^m_{b,2})\right.\right\} \text{ and } \underline{a_2} = \left\{a_2 \left| V^m_{b,2}(\underline{y}^m_{b,2}) \geq V^k_{b,2}(\underline{y}^m_{b,2})\right.\right\}.$$

The two thresholds, $\overline{a}_2$ and $\underline{a}_2$, divide the debt space into three regions. The first region is the safe region in which the borrowing country never defaults. The second region is the risky region with a non-zero default probability. The borrowing country defaults depending on the income realization in period $t = 2$. The last region is the default region where the borrowing country always defaults. Hence,

$$\delta_2 = \begin{cases} 0 & \text{if} \quad a_2 \leq \underline{a}_2, \\ (0,1) & \text{if} \quad \overline{a}_2 > a_2 > \underline{a}_2, \\ 1 & \text{if} \quad a_2 \geq \overline{a}_2. \end{cases}$$

Given the bond price the borrowing country will borrow an amount in order to smooth its consumption across periods. We derive the first order condition of the borrowing country with logarithmic preferences.[18] In this case, the debt supply function of the borrowing country is given by,

$$q_2 = \begin{cases} \frac{\beta_b\left(y^m_{b,1}-a_1\right)E_1\left[\frac{1}{y^m_{b,2}-a_2}\right]}{1-\beta_b a_2 E_1\left[\frac{1}{y^m_{b,2}-a_2}\right]} & \text{if} \quad a_2 \leq \underline{a}_2, \\ \\ \frac{\beta_b\left(y^m_{b,1}-a_1\right)\int_{\widehat{y}}^{\overline{y}^m_{b,2}} \frac{1}{y^m_{b,2}-a_2} dF_{y^m_{b,2}}}{1-\beta_b a_2 \int_{\widehat{y}}^{\overline{y}^m_{b,2}} \frac{1}{y^m_{b,2}-a_2} dF_{y^m_{b,2}}} & \text{if} \quad \overline{a}_2 > a_2 > \underline{a}_2, \\ \\ 0 & \text{if} \quad a_2 \geq \overline{a}_2. \end{cases} \tag{8}$$

The bond price function in period t **=1** For a given debt, the lending country with logarithmic preferences chooses a price which maximizes Eq (5) subject to q_2 being determined by Eq (8). The first order condition of the lender's maximization problem is,

$$\frac{\frac{\partial q_2}{\partial a_2} a_2 + q_2}{y^m_{l,1} + a_1 - a_2 q_2} = \begin{cases} \beta_l E_1\left[\frac{1}{y^m_{l,2}+a_2}\right] & \text{if} \quad a_2 \leq \underline{a}_2, \\ \\ \beta_l E_1\left[\frac{(1-\delta_2)}{y^m_{l,2}+a_2(q_2^*)} - \delta_2'(V^m_{l,2} - V^k_{l,2})\right] & \text{if} \quad \overline{a}_2 > a_2 > \underline{a}_2, \\ \\ 0 & \text{if} \quad a_2 \geq \overline{a}_2. \end{cases} \tag{9}$$

3.3 Equilibrium

Definition: *A monopolistic equilibrium with two countries–a lender and a borrower–is defined as a set of policy functions for debt supply,* $a_2(q_2)$*; default decision,* $\widehat{\delta}_2(q_2)$*; price of debt,* q_2*; each agents' consumption in every period for a specific monetary regime,* $\left\{c^m_{l,1}, c^m_{l,2}, c^m_{l,3}, c^k_{l,3}, c^m_{b,1}, c^m_{b,2}, c^m_{b,3}, c^k_{b,3}\right\}$*, such that:*

[18] We apply the Leibniz's rule for deriving the F.O.C. in the risky region. Note that $V^m_{b,2}(\widehat{y}) = V^k_{b,2}(\widehat{y})$.

(i) *Taking default decision and debt supply,* $\left\{\widehat{\delta}_2(q_2),\, a_2(q_2)\right\}$ *as given, price of debt,* $\{q_2\}$ *solves lender's maximization problem in period* $t = 1$.

(ii) *Taking price of debt,* $\{q_2\}$ *as given, the default decision,* $\left\{\widehat{\delta}_2(q_2)\right\}$ *at time* $t = 2$ *and the choice of debt supply,* $\{a_2(q_2)\}$*, at time* $t = 1$ *maximizes borrower's utility in period* $t = 1$.

(iii) *The resource constraints of the economy are satisfied every period.*

3.4 An Example

The example in this section simplifies the full model and uses a stylized economy to illustrate the key results of the model. All the features of the full model are retained but two. First, the continuation values associated with the future are now restricted to only period $t = 3$. Second, the income shocks of the lender and the borrower are independently and identically distributed (i.i.d.) with two income states. We further assume that the lender and the borrower are risk averse with CRRA preferences and receive lower income state shock in period $t = 1$.

We set the values to parameters in our example as follows. The risk aversion parameters, σ_b and σ_l, of the borrower and the lender are assumed to be 3 and 2, respectively. The default output loss is set around 5%.[19] The lender and the borrower are assumed to be equally patient with the discount factor, $\beta_b = \beta_l = 0.98$. The initial debt level is a_1 =1.3. The two state income shocks for the monetary union and the standalone countries are set at $\left\{\underline{y}^m, \overline{y}^m\right\} = \{4, 12\}$ and $\left\{\underline{y}^k, \overline{y}^k\right\} = \{4.5, 5\}$, respectively. We assume that the income structure is same for the lender and the borrower. In both of the monetary regimes, the probability of receiving a low income shock is 1%.

Figure 15 plots the debt supply as a function of the bond price and also depicts the corresponding equilibrium price and quantity (represented by the black dot). The three shaded regions are associated with repayment and default. The light grey region corresponds to the safe region and debt issued within this range is always repaid. The middle region is the risky region with default risk. The last and the dark grey region is the default region. The borrowers supply of debt

[19] In literature this may vary from 1-5%. Conesa and Kehoe (2012).

is increasing in the bond price within the safe and risky region. However, in the default region the borrower always supplies maximum amount of debt. The equilibrium debt and price for the given parameter space is $\{a_2^*, q_2^*\} = \{3.71, 0.0392\}$. As we increase the initial income of the lender from the low state to the high state, the new equilibrium shifts to $\{a_2^*, q_2^*\} = \{3.75, 0.0399\}$, depicted by the grey dot. The default probability remains same for both the equilibriums, which is the probability of receiving the low income shock, and does not depend on the debt supplied due to i.i.d. income structure. Further, we also evaluate the scenario when the two countries are standalone, where $\{\underline{y}^m, \overline{y}^m\} = \{\underline{y}^k, \overline{y}^k\}$, and assess the debt market as compared to when they are in the monetary union. In this example we find that under the assumption of standalone countries the two economies do not borrow or lend to each other.

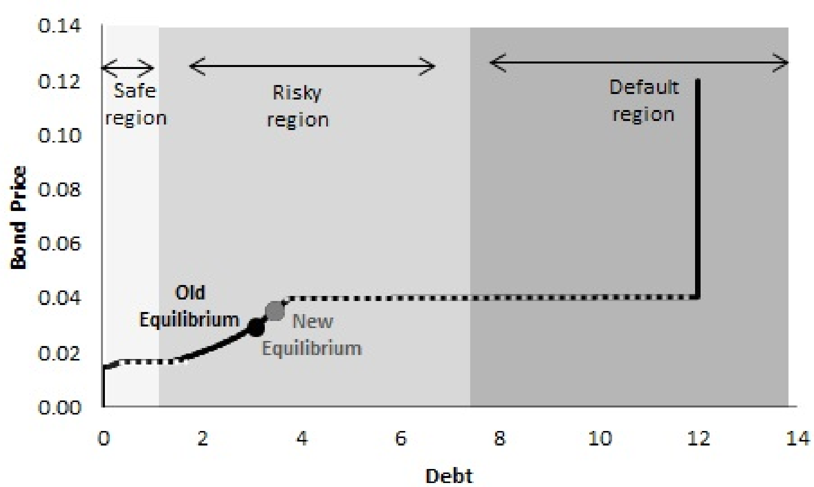


Figure 15: Debt supply and Equilibrium

3.5 Monetary union vis-à-vis non-Monetary union

We are interested in comparing the borrowing and lending behaviour in two monetary regimes: Monetary union and non-Monetary union. The first regime refers to the one described above where in period $t = 1$ the countries are in the monetary union. The income of the member countries incorporates both the benefits and the costs of being in the monetary union. In the second regime, countries are not the members of the monetary union in period $t = 1$ and follow the income processes of a standalone.

In order to analyse the borrowing and lending behaviour, we solve the model numerically under a specific parameter space as described in Table 2. We assume that the lending and the borrowing countries have logarithmic preferences. We follow the literature to assign values to the parameters in the model, β_b, β_l, ρ_b, ρ_l and γ. In this economy, we assume that the income trend and volatility are same for the borrowing and the lending countries when in a particular monetary regime. The borrowing country's initial income and debt are set such that it has positive initial wealth, $y^m_{b,1} - a_1 > 0$. We assume that the initial income of the lending country is higher than the initial income of the borrowing country.

When the default output cost is low, as in Table 2, a standalone borrowing country supplies either no or very high debt. As a result, we observe no borrowing and lending in equilibrium. For a very high default output cost, at 70%, the standalone borrowing country supplies a positive amount of debt (Figure 16). The former case illustrates that the monetary union by itself promote borrowing and lending amongst the member countries. The latter case, even though unrealistic, strengthens the above argument and emphasizes the key role that the monetary union plays, i.e. it facilitates borrowing and lending with lower debt yields even though the default probability is higher (refer to Result 2).

Parameter	Description	Value
α^m	Income trend (MU)	3%
α^k	Income trend (non MU)	2%
ς^m	Income volatility (MU)	8%
ς^k	Income volatility (non MU)	2%
$\mathrm{y}^m_{1,b}$	Borrower's initial income	5
$\mathrm{y}^m_{1,l}$	Lender's initial income	8
γ	Default output cost	1%
ρ_b	Borrower's income persistence	0.98
ρ_l	Lender's income persistence	0.98
β_b	Borrower's discount factor	0.7
β_l	Lender's discount factor	0.98
a_1	Initial debt	4.8

Table 2: Baseline economy

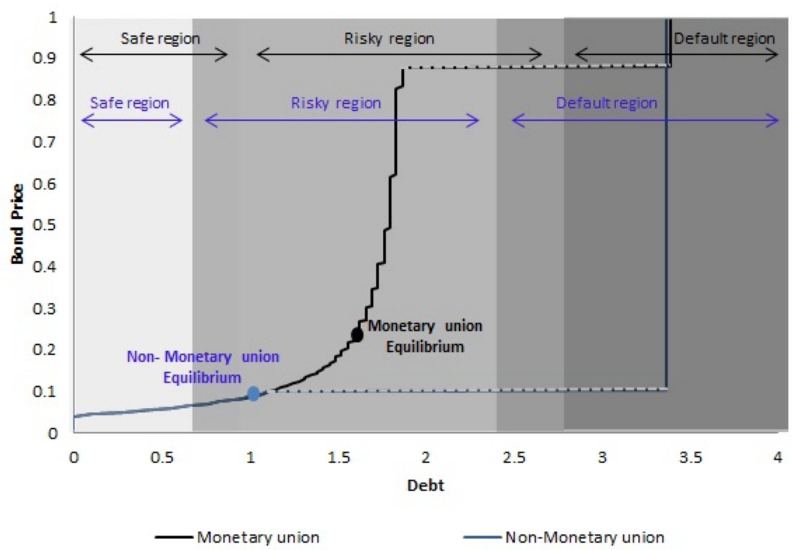


Figure 16: Debt supply with high default output cost

4 A cross country income disparity

We now turn our attention to another important question, which we address in this paper: Does forming the monetary union with high initial income disparity between the member countries influences the debt market equilibrium outcome?

We use the baseline economy as reported in Table 2. We change the income trend parameter in the monetary union to $\alpha^m = 6\%$ for illustrative purpose. In the first step, we calculate the debt supply of the borrowing country for a given price and then compute the equilibrium. Later, we increase the initial income disparity between the member countries of the monetary union. It is worth stating that the debt supply will remain unchanged if we change the parameters specific to the lending country. In the new equilibrium, the richer lending country buys higher amount of debt issued at lower yield from the borrowing country.

Further, we also find that the lending country with a higher discount factor, β_l, and a lower income persistence, ρ_l, has similar effects as with an increase in initial income. Figure 17 plots the debt supply of the borrowing country and corresponding debt issuance and its price in the old (represented by the black dot) and the new (represented by the grey dot) equilibria.

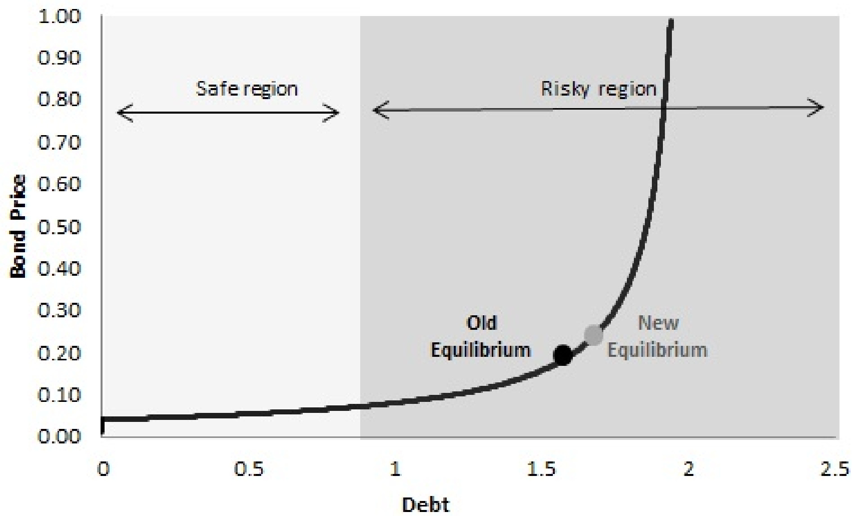


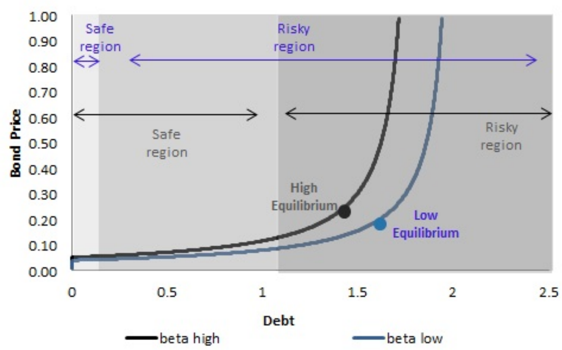
Figure 17: Comparative statics

5 Comparative statics

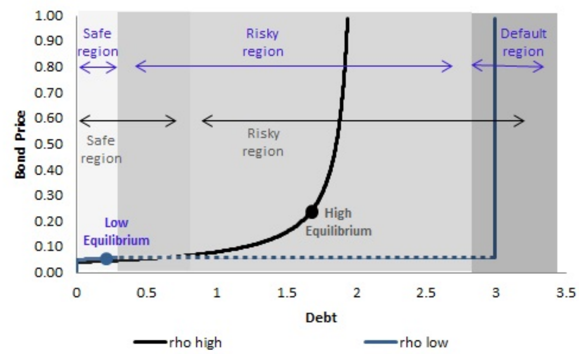
We perform additional comparative static exercises with respect to the borrowing country. We change the discount factor, β_b, and the income persistence, ρ_b, of the borrowing country. A more impatient borrowing country supplies a higher amount of debt at all levels of price. In equilibrium, higher amount of debt is issued at a lower price. Figure 18 plots the debt supply and equilibria associated with the change in the borrowing country's discount factor in panel (a).

An increase in borrowing country's income persistence leads to lower debt supply for higher price and higher debt supply for lower prices. In equilibrium, both debt issuance and its price increase. However, the new equilibrium has associated default risk. These changes are illustrated in panel (b) of figure 18.

We also look at the effect of change in initial indebtedness of the borrower. A higher initial debt increases the amount of debt supplied by the borrower. In equilibrium, the amount of debt issued increases but the bond price reduces. This result is in line with the theoretical literature, as higher initial debt is associated with higher borrowing in the future and has a 'debt roll-over' effect. The cost of borrowing may not necessarily go down as a highly indebted country poses higher risk of default. Figure 19 plots the debt supplies and respective equilibria.



(a) Discount factor



(b) Income persistence

Figure 18: Comparative statics

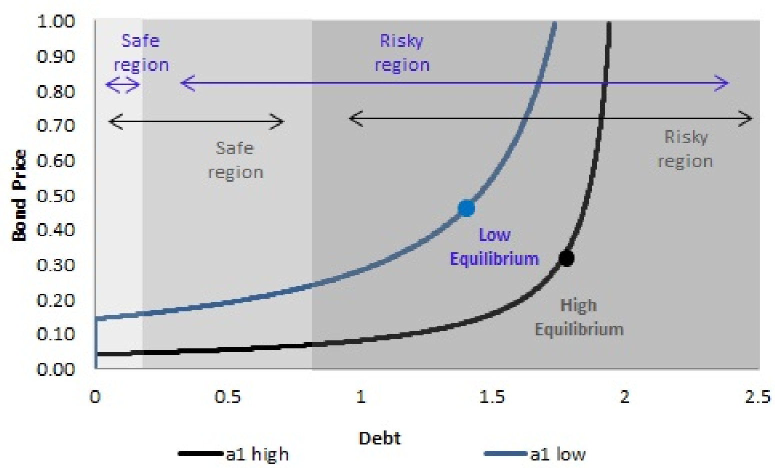


Figure 19: Initial Indebtedness

6 Concluding Remarks

A monetary union entails both costs and benefits for its member countries and has profound implications for the overall global economy. The recent debt crisis in the peripheral economies of the EMU has highlighted several vulnerabilities of the union. Our paper explores two main questions: Does being in the monetary union change the way its debt market evolves? Further, does higher initial income differential between member countries affect the amount of debt issuance, its price and incentives to default?

We develop a model of lending and borrowing in the monetary union, which is based on the endogenous sovereign debt default literature. We have two innovations in our framework. First, we introduce a reduced form representation of the monetary union and second, the lender is assumed to be risk averse with monopoly power. These key assumptions are based on the findings from several papers and on our own calculations with data from the IMF's CPIS and OECD database.

A monetary union provides another layer of trade-off associated with default in addition to the standard output default cost. In the event of default, the borrower has to forgo not only a fraction of the output as penalty but also future benefits associated with the union. However, it also leads to reduction in income volatility due to discretion over the monetary policy. Similarly, the default affects the lender in more than one way. The lender cannot collect the outstanding amount of debt and faces the consequences of a breakup of the monetary union.

We find that monetary union facilitates existence of the debt market and allows its member countries to borrow and lend more at lower cost of borrowing under circumstances when it is not possible do so if the countries were standalones. We also observe in the stylized economy that when the initial income difference grows between the lending and the borrowing member countries, debt issuance increases and debt yields drop. We illustrate that similar effects are observed when the lender has incentives to save for future and smooth consumption inter-temporally, for e.g. with higher discount factor and lower income persistence. A higher initial income not only allows lender to save more today but also increase the future benefits of staying in the union. Both higher discount factor and lower income persistence boost saving and mitigate credit rationing. The lender is able to receive higher monopoly rent today by buying more debt, which is issued by the borrower, at lower rate of interest and also reap benefits of staying in the union.

The changes in borrowing country specific features not only shift the debt supply but also influence the equilibrium outcome. The borrowing country supplies higher debt when it is more impatient or indebted. Since these are conducive for default, higher debt is issued at higher cost of borrowing in equilibrium. For lower income persistence of the borrower, safe but costlier debt is issued in equilibrium.

Our model is simple but it has interesting implications for the monetary union. It confirms the growing interest in forming monetary union, as was the case of the EMU, from the lens of the debt market evolution. It also shows that interplay between the attributes of the member countries in the monetary union is crucial in determining the equilibrium of the debt market. We conclude that an empirical exercise along the lines of our model will help future research shed more light on the debt market in the monetary union. However, this will require more frequently reported and longer time series data.

References

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *The American Economic Review*, 690–712.

Baldwin, R. E. (2006). The euro's trade effects.

Conesa, J. C. and T. J. Kehoe (2012). Gambling for redemption and self-fulfilling debt crises. Technical report, Federal Reserve Bank of Minneapolis.

CPIS, I. T. F. (2013). *Coordinated portfolio investment survey, **IMF T**ask **F**orce.* International Monetary Fund.

De Grauwe, P. (2012). *Economics of monetary union.* Oxford University Press.

Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies 48*(2), 289–309.

Flam, H. and H. Nordström (2006). Trade volume effects of the euro: Aggregate and sector estimates.

Frankel, J. and A. Rose (2002). An estimate of the effect of common currencies on trade and income. *The Quarterly Journal of Economics 117*(2), 437–466.

Fratzscher, M. and L. Stracca (2009). The political economy under monetary union: Has the euro made a difference? *Economic Policy 24*(58), 307–348.

Lane, P. R. (2012). The european sovereign debt crisis. *The Journal of Economic Perspectives 26*(3), 49–67.

Luque, J., M. Morelli, and J. Tavares (2014). A volatility-based theory of fiscal union desirability. *Journal of Public Economics 112*, 1–11.

Micco, A., E. Stein, and G. Ordoñez (2003). The currency union effect on trade: early evidence from emu. *Economic policy 18*(37), 315–356.

OECD (1999). European **M**onetary **U**nion facts, challenges and policies.

Papaioannou, E. and R. Portes (2008). Costs and benefits of running an international currency. Technical report, Directorate General Economic and Monetary Affairs (DG ECFIN), European Commission.

Shambaugh, J. C., R. REIS, and H. REY (2012). The euro's three crises [with comments and discussion]. *Brookings Papers on Economic Activity*, 157–231.

Werning, I. and E. Farhi (2012). Fiscal unions. Technical report, National Bureau of Economic Research.

A Appendix

A.1 Net lending to Spain by Japan, Norway, the USA and the UK

Japan, Norway, the UK and the USA formed the third biggest group of lenders owning Spanish debt from 2001-2013. Japan and Norway were net lenders throughout 2001-2013, while Spain was the net lender to the USA until 2010. The UK remained a net lender of Spain post-2004 and increased lending after a decline during the financial crises of 2007-2009. The net lending increased in the first sub-period from USD 10 bn in 2001 to USD 29 bn for Japan; from USD 3 bn in 2001 to USD 20 bn for Norway and from USD 7 bn in 2001 to USD 20 bn for the UK. It dropped by 22% for Japan; by 100% for Norway and increased by 136% for the UK in the second sub-period. The growth patterns in net lending had similarities between the UK and Norway, which are members of the European Union, with those of the lending Euro countries.

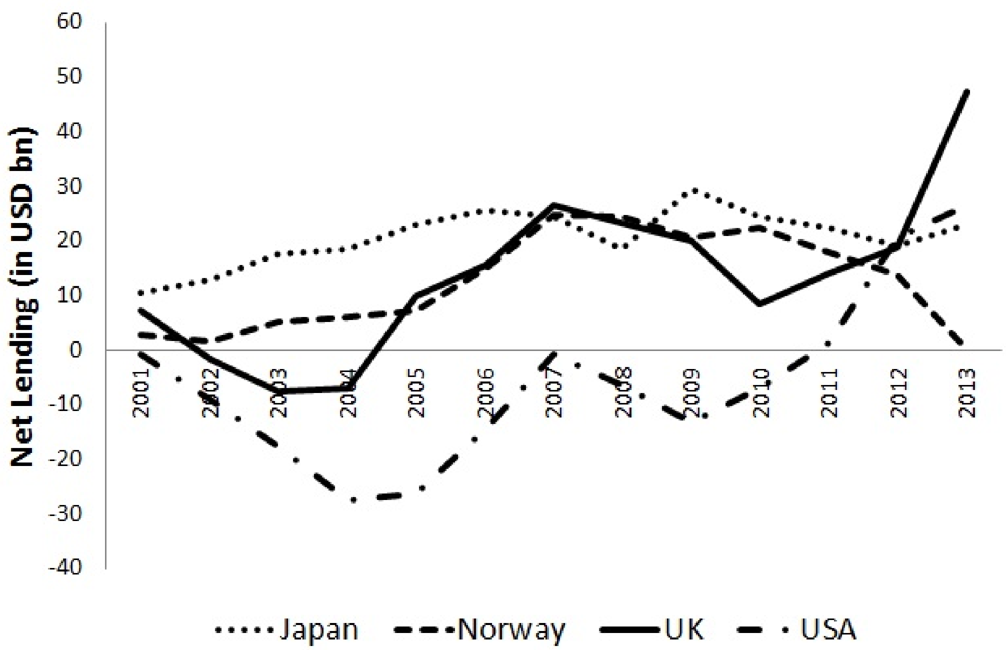


Figure A.1: Net Lending

Source: Quarterly External Debt Statistics, Reinhart and Rogoff.

A.1.1 Spanish debt securities holder's profile

Year	Financial Sector	General government	Non-Financial Sector
2013	90.3%	6.6%	3.1%
2012	90.0%	7.3%	2.8%
2011	92.8%	5.3%	1.9%
2010	90.5%	8.1%	1.4%
2009	98.9%	0.1%	1.0%
2008	99.1%	0.0%	0.8%
2007	99.0%	0.0%	0.8%

Table A.1: Germany

Source: Coordinated Portfolio Investment Survey (IMF).

Financial Sector:Central Bank, Deposit-taking Corporations, Other Financial Corporations; Non-Financial Sector: Nonfinancial Corporations, Households and Non-profit institutions serving households.

Year	Financial Sector	General government	Non-Financial Sector
2013	96.0%	0.8%	3.2%
2012	95.3%	0.6%	4.1%
2011	98.3%	0.8%	0.9%
2010	97.7%	1.3%	0.9%
2009	97.7%	0.8%	1.5%
2008	96.8%	0.9%	1.5%
2007	96.6%	0.8%	2.6%

Table A.2: France

Source: Coordinated Portfolio Investment Survey (IMF).
Financial Sector:Central Bank, Deposit-taking Corporations, Other Financial Corporations; Non-Financial Sector: Nonfinancial Corporations, Households and Non-profit institutions serving households.

A.2 Proofs

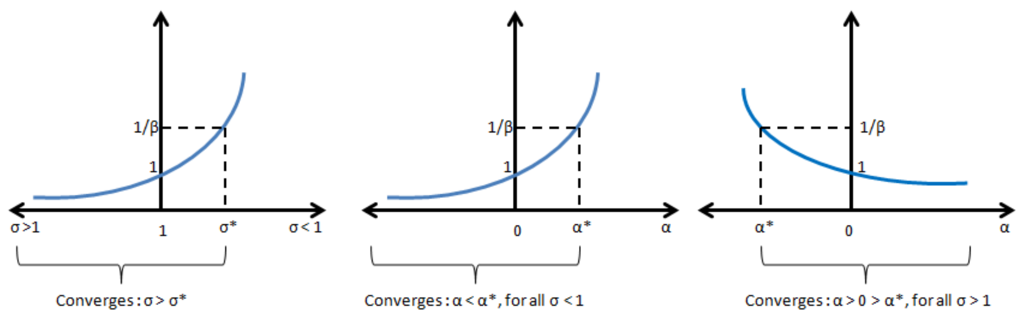
Continuation Value for period 3 In this section, we define the functional form of the continuation value with CRRA preferences. If the borrowing country chooses to repay (or default) its debt in period $t = 2$, it recieves income shocks associated with a monetary union (or standalone country) in period $t \geq 3$. So the continuation values when at period $t = 2$, for a given i and j, can be evaluated as follows,

$$\widetilde{V} = E_2\left[u(c_3) + \beta u(c_4) + \beta^2 u(c_5) + ...\right] = \frac{S - \frac{1}{1-\beta}}{1-\sigma}$$

<u>Proof of Result 1:</u>

In the above equation, $\mathcal{S}$ represents the infinite sum of the sequence, s_n,

$S = \sum_{n=1}^{\infty} s_n$, where $s_n = \beta^{n-1} y_2^{(1-\sigma)\rho^n} e^{\frac{(1-\rho^{2n})(1-\sigma)^2 \varsigma^2}{2(1-\rho^2)}} e^{\frac{(-3\rho^{n-1}+\frac{n+2}{\rho}-\frac{1-\rho^{n-1}}{1-\rho})(1-\sigma)\alpha\rho}{1-\rho}}$. For the convergence of the above infinite series, as per the 'Ratio test' and using the property that $\rho_b < 1$, $\beta_b < 1$ and $\rho^n \to 0$ as $n \to \infty$, we need that $\frac{s_{n+1}}{s_n} \to \beta e^{\frac{\alpha(1-\sigma)}{1-\rho}} < 1$ as n $\to \infty$. This shall be true for a positive growth, $\alpha > 0$ (since persistence, $\rho < 1$) iff $\sigma \geq \widetilde{\sigma}$, where, $\widetilde{\sigma} < 1$. Conversly, we may think that for all $\sigma < 1$ we need $\alpha < \alpha * (> 0)$ and for $\sigma > 1$ we need $\alpha > \alpha * (< 0)$



The continuation value for the benchmark case with logarithmic preferences is given by,

$$\widetilde{V} = E_2 \left[log(y_3) + \beta log(y_4) + \beta^2 log(y_5) + ... \right]$$

$$= \left[\frac{\rho log(y_2)}{1-\beta\rho} + \frac{3\alpha}{1-\beta\rho} + \frac{4\beta\alpha}{1-\beta\rho} + \frac{5\beta^2\alpha}{1-\beta\rho} + ... \right]$$

$$= \left[\frac{\rho log(y_2)}{1-\beta\rho} + \frac{\alpha}{1-\beta\rho} \frac{3-2\beta}{(1-\beta)^2} \right]$$

Decision to Default/Repayment at Period $t = 2$ For CRRA preferences and for a given debt, a_2, the borrowing country decides to repay the loan in period $t = 2$ depending on the following condition:

$$(y_{b,2}^m - a_2)^{1-\sigma_b} - ((1-\gamma) y_{b,2}^m)^{1-\sigma_b} \geq \beta_b (1-\sigma_b) \widetilde{\Omega} \qquad (10)$$

where $\widetilde{\Omega}_b = \frac{S^d - S^r}{1-\sigma}$

When the borrower has logarithmic utility,

$$\widetilde{\Omega}^m = \frac{\rho_b(log(y_2^k)-log(y_2^m))}{1-\beta_b\rho_b} + \frac{\alpha_b^k-\alpha_b^m}{1-\beta_b\rho}\frac{3-2\beta_b}{(1-\beta_b)^2}$$

We look at the properties of the function $\mathcal{H}(y_{b,2}^m, a_2) = V_{b,2}^m(a_2) - V_{b,2}^k$. This is an increasing function of $y_{b,2}^m$ and we show it below.

$$\mathcal{H}(y_{b,2}^m, a_2) = log(y_{b,2}^m - a_2) + \beta_b\widetilde{V}_b^m - log((1-\gamma)y_{b,2}^m) - \beta_b\widetilde{V}_b^k$$

$$= log(\frac{y_{b,2}^m - a_2}{(1-\gamma)y_{b,2}^m}) - \beta_b(\widetilde{V}_b^k - \widetilde{V}_b^m)$$

Let $\mathcal{D} = \frac{\beta_b\rho_b}{1-\beta_b\rho_b} > 1$ and $\mathcal{C} = \frac{(3-2\beta)(\alpha^k-\alpha^m)}{(1-\beta\rho)(1-\beta)^2}$.

$$= -log((1-\gamma)) + log(y_{b,2}^m - a_2) - log(y_{b,2}^m) + log((y_{b,2}^m)^{\mathcal{D}}) - log((y_{b,2}^k)^{\mathcal{D}}exp(\mathcal{C}))$$

Notice that $\frac{\partial\mathcal{H}}{\partial y_{b,2}^m} = \frac{a_2}{y_{b,2}^m - a_2} + \frac{\mathcal{D}}{y_{b,2}^m} > 0$.

Proof of Proposition 1:

We provide evidence in support of the above theorem by considering specific cases when $\sigma \in \{0.5, 1, 2\}$. In doing so we seperate the default and repayment sets based on the risk appetite of the borrower.

For logarithmic preferences, when $\sigma = 1$,

$$\widehat{a}_2 = y_2^m\left(1-(1-\gamma)\exp\left(\beta_b\left[\frac{\rho_b(log(y_2^k)-log(y_2^m))}{1-\beta_b\rho_b} + \frac{\alpha_b^k-\alpha_b^m}{1-\beta_b\rho}\frac{3-2\beta_b}{(1-\beta_b)^2}\right]\right)\right)$$

First order derivative of $\widehat{a}_2$ w.r.t to the parameters under consideration (ceteris paribus) are

$$\frac{\partial\widehat{a}_2}{\partial\alpha_b^m} = \frac{\beta_b(1-\gamma)y_2^m(3-2\beta_b)\exp\left(\beta_b\widetilde{\Omega}^m\right)}{(1-\beta_b\rho_b)^3(1-\beta_b)^2} \geq 0$$

$$\frac{\partial\widehat{a}_2}{\partial\alpha_b^k} = -\frac{\beta_b(1-\gamma)y_2^m(3-2\beta_b)\exp\left(\beta_b\widetilde{\Omega}^m\right)}{(1-\beta_b\rho_b)^3(1-\beta_b)^2} \leq 0$$

$$\frac{\partial\widehat{a}_2}{\partial y_2^k} = -\frac{\beta_b(1-\gamma)\rho_b y_2^m\exp\left(\beta_b\widetilde{\Omega}^m\right)}{y_2^k(1-\beta_b\rho_b)} \leq 0,$$

$\frac{\partial \widehat{a}_2}{\partial y_2^m} = 1 + (1-\gamma)\exp\left(\beta_b \widetilde{\Omega}^m\right) \frac{2\beta_b\rho_b - 1}{1-\beta_b\rho_b} \geq 0$ if $\beta_b\rho_b \geq \frac{1}{2}$ else it is ambigous. We know $\beta_b < 1$ and applying this to our condition we get $\rho_b \geq \frac{1}{2\beta_b} > \frac{1}{2}$. This remains true under the parameter space that we consider.

$\frac{\partial \widehat{a}_2}{\partial \gamma} = y_2^m \exp\left(\beta_b \widetilde{\Omega}^m\right) \geq 0,$

Next we consider the case for CRRA preferences where $\sigma > 1$. The borrowers' indifference between default and repayment arises when

$$(y_2^m - a_2)^{1-\sigma} - ((1-\gamma)y_2^m)^{1-\sigma} = \beta(1-\sigma)\widetilde{\Omega}_b \tag{11}$$

here $(1-\sigma)\widetilde{\Omega}_b = S^D - S^R$ and $S = \sum_{n=1}^{\infty} \beta^{n-1} y_2^{(1-\sigma)\rho^n} e^{\frac{(1-\rho^{2n})(1-\sigma)^2 \varsigma^2}{2(1-\rho^2)}} e^{\frac{(-3\rho^{n-1} + \frac{n+2}{\rho} - \frac{1-\rho^{n-1}}{1-\rho})(1-\sigma)\alpha\rho}{1-\rho}}$

Thus,the debt level which shall make the borrower indifferent between repayment or default is given by:

$\Rightarrow \widehat{a}_2 = y_2^m - \left[((1-\gamma)y_2^m)^{1-\sigma} + \beta_b(1-\sigma)\widetilde{\Omega}_b\right]^{\frac{1}{1-\sigma}}$

Let, $C_b = \frac{(1-\sigma)^2(1-\rho_b^{2n})}{2(1-\rho_b^2)} > 0$ and $Z_b = \frac{(-3\rho_b^{n-1} + \frac{n+2}{\rho_b} - \frac{1-\rho_b^{n-1}}{1-\rho_b})(1-\sigma)\rho_b}{1-\rho_b} > 0$ iff $\sigma < 1$.

An analysis of the derivatives is presented below for the case when $\sigma = 2$. However, this may be extended to the general case where $\sigma_b > 1$ and the results hold true. In the special case with $\sigma_b = 2$

$\widehat{a} = y_2^m - \frac{1}{\frac{1}{(1-\gamma)y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}} \right)}$

The first order derivative of a_2 are given below. Of course, the expressions below may not be determined or are discontinous if the denominator goes to zero. However, it is worth stating that the denominator is always non zero in the parameter and state space we conider.

$$\frac{\partial \widehat{a}_2}{\partial \alpha_b^m} = -\frac{\sum_{n=1}^{\infty} \beta^n Z_b \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}}{\left[\frac{1}{(1-\gamma) y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \geq 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \alpha_b^k} = \frac{\sum_{n=1}^{\infty} \beta^n Z_b \frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}}}{\left[\frac{1}{(1-\gamma) y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \leq 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial y_2^m} = 1 + \frac{-\frac{1}{(1-\gamma) y_2^{m2}} + \sum_{n=1}^{\infty} \beta^n \rho_b^n \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{1+\rho_b^n}}}{\left[\frac{1}{(1-\gamma) y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \geq 0$$ in the parameter space under consideration.

$$\frac{\partial \widehat{a}_2}{\partial y_2^k} = -\frac{\sum_{n=1}^{\infty} \beta^n \rho_b^n \frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{1+\rho_b^n}}}{\left[\frac{1}{(1-\gamma) y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \leq 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \varsigma_b^{2k}} = \frac{\sum_{n=1}^{\infty} \beta^n C_b \frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}}}{\left[\frac{1}{(1-\gamma) y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \geq 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \varsigma_b^{2m}} = -\frac{\sum_{n=1}^{\infty} \beta^n C_b \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}}{\left[\frac{1}{(1-\gamma) y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \leq 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \gamma} = \frac{1}{(1-\gamma)^2 y_2^m \left[\frac{1}{\gamma y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \geq 0, \text{ unambigously.}$$

(Q.E.D)

Proof of Proposition 2:

Let, $y_{b,2}^m = y_{b,2}^k = y_{b,2}$.

$$\Rightarrow \beta_b \widetilde{\Omega}_b = \frac{\beta_b \rho_b (log(y_{b,2}) - log(y_{b,2}))}{1-\beta_b \rho_b} + \frac{\beta_b \rho_b (3-2\beta_b)(\alpha_b^k - \alpha_b^m)}{(1-\beta_b \rho_b)(1-\beta_b)^2} = \frac{\beta_b \rho_b (3-2\beta_b)(\alpha_b^k - \alpha_b^m)}{(1-\beta_b \rho_b)(1-\beta_b)^2} \leq 0$$

$$\Rightarrow \exp(\beta_b \widetilde{\Omega}_b) \leq 1$$

$$\Rightarrow y_{b,2}\left(1-(1-\gamma)\exp(\beta_b \widetilde{\Omega}_b)\right) \geq y_{b,2}\left(1-(1-\gamma)\right)$$

$$\Rightarrow \widehat{a}_2^m \geq \widehat{a}_2^k$$

(Q.E.D)

Proof of Corollory:

We can write that $\widehat{a}_2^m = \widehat{a}_2^k = a_2$ iff
(i) $y_b^{*m} \geq y_b^{*k}$ and default penalty is big and trend growth difference is high for different regimes or when
(ii) $y_b^{*m} \leq y_b^{*k}$.

$$\widehat{a}_2^m : V_{b,2}^m(a_2) = V_{b,2}^k \text{ and } \widehat{a}_2^k : V_{b,2}^s(a_2) = V_{b,2}^k$$

$$\Rightarrow \gamma y_{b,2}^k = y_{b,2}^m\left(1-(1-\gamma)exp\left[\beta_b(\widetilde{V}^k - \widetilde{V}^m)\right]\right)$$

$$\Rightarrow \gamma \frac{y_b^{*k}}{y_b^{*m}} = 1-(1-\gamma)\left(\frac{y_{b,2}^k}{y_b^{*m}}\right)^{\mathcal{D}} exp(\mathcal{C})$$

$$\Rightarrow \gamma \frac{y_b^{*k}}{y_b^{*m}} + (1-\gamma)exp(\mathcal{C})\left(\frac{y_{b,2}^k}{y_b^{*m}}\right)^{\mathcal{D}} = 1$$

where, $\mathcal{D} = \frac{\beta\rho}{1-\beta\rho} > 1$ and $\mathcal{C} = \frac{\beta(3-2\beta)(\alpha^k-\alpha^m)}{(1-\beta\rho)(1-\beta)^2} < 0$

Note that the above equality will not be satified for the cases when, $y_{b,2}^k < y_b^{*k} < y_b^{*m}$ and $y_b^{*k} < y_{b,2}^k < y_b^{*m}$. The above will be satisfied for a higher γ or a higher difference between α^m and α^m when $y_b^{*k} < y_b^{*m} < y_{b,2}^k$. Thus, if we have a small default penalty, we should have $y_b^{*k} > y_b^{*m}$.

Further, if we compare the case when the borrower recieves same shocks for the autarky when in either of the regimes we get,

$$\gamma \frac{y_b^k}{y_b^m} + (1-\gamma)exp(\mathcal{C})\left(\frac{y_b^k}{y_b^m}\right)^{\mathcal{D}} = 1$$

The above equation will never be satisfied if $\frac{y_b^k}{y_b^m} < 1$ or $y_b^k < y_b^m$, since γ, $1-\gamma$ and $exp(\mathcal{C}) < 1$. However, it is possible for the equality to hold for $\frac{y_b^k}{y_b^m} > 1$ or $y_b^k > y_b^m$.

(Q.E.D)

Default probability in period 1

Proof of Result 2:

In order to prove this result we make use of the properties of the error function.

Since, $-1 \leq \text{erf}(.) \leq 1$

$\Rightarrow 0 \leq 1 - \text{erf}(.) \leq 2 \Rightarrow 0 \leq \frac{1-\text{erf}(.)}{2} \leq 1$

$\Rightarrow 0 \leq \delta_2 \leq 1$.

Again, given $a_2 = y_b^* \left(1 - (1-\gamma)\exp\left(\beta_b \left[\frac{\rho_b(log(y_{b,2}^k) - log(y_b^*))}{1-\beta_b\rho_b} + \frac{\alpha_b^k - \alpha_b^m}{1-\beta_b\rho} \frac{3-2\beta_b}{(1-\beta_b)^2}\right]\right)\right)$ we take the derivative with respect to a_2 to obtain,

$1 = \frac{\partial y_b^*}{\partial a_2}\left[1 - (1-\gamma)\exp(\beta_b\widetilde{\Omega}_b) + \frac{\beta_b\rho_b}{1-\beta_b\rho_b}(1-\gamma)\exp(\beta_b\widetilde{\Omega}_b)\right]$

$\Rightarrow \frac{\partial y_b^*}{\partial a_2} = \frac{1}{\left[(1-(1-\gamma)\exp(\beta_b\widetilde{\Omega}_b))+\frac{\beta_b\rho_b}{1-\beta_b\rho_b}(1-\gamma)\exp(\beta_b\widetilde{\Omega}_b)\right]} = \frac{1}{1+(1-\gamma)\exp(\beta_b\widetilde{\Omega}_b)\frac{2\beta_b\rho_b-1}{1-\beta_b\rho_b}} \geq 0$ if $\rho_b\beta_b \geq \frac{1}{2}$, which shall be the case in our setup.

$$-\delta_2'(a_2) = \frac{-1}{2y_{b,2}^*\varsigma_b^m}\sqrt{\frac{2}{\pi}}\left[exp\left(-\frac{log^2\left(\frac{y_{b,2}^*}{(y_{b,1}^m)^{\rho_b}\exp(2\alpha_b^m)}\right)}{2(\varsigma_b^m)^2}\right)\right]\frac{\partial y_{b,2}^*}{\partial a_2} < 0. \qquad (12)$$

where, $\frac{\partial y_{b,2}^*}{\partial a_2} \geq 0$ for all the values of σ.
(Q.E.D)

Debt threshold changes as the risk aversion of the borrower changes (refer to fig A.2).

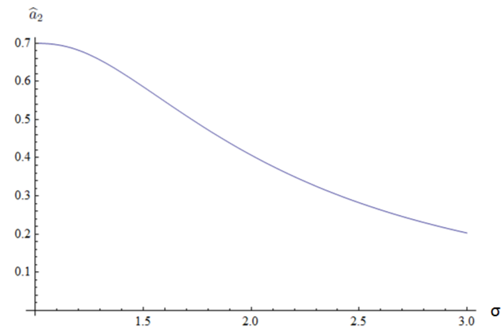


Figure A.2: 'Indifference' debt v.s. risk aversion

Proof of Proposition 3:

Let us say, $y^m_{b,1} = y^k_{b,1}$. We already know that $y^{*k}_b = \frac{a_2}{\gamma}$ and $(y^{*m}_b)^{\mathcal{B}-1}(y^{*m}_b - a_2) = (1-\gamma)(y^k_b)^{\mathcal{B}} e^{\mathcal{C}(\alpha^k - \alpha^m)} \geq 0 \Rightarrow y^{*m}_b \geq a_2$.

Thus, we are in one of the two cases, (i) $a_2 \leq y^{*m}_b \leq \frac{a_2}{\gamma} = y^{*k}_b$ or (ii) $a_2 \leq \frac{a_2}{\gamma} = y^{*k}_b \leq y^{*m}_b$. For an debt level a_2, $y^{m*}_{b,2}(a_2) \leq y^{k*}_{b,2}(a_2)$ whenever $\gamma \mapsto \underline{\gamma}$ (where $\underline{\gamma}$ is close to zero). In this scenario we can say, since the error and logarithmic functions are monotonically increasing, that

$$\frac{y^{m*}_{b,2}}{(y^m_{b,1})^{\rho_b}\exp(2\alpha^m_b)} \leq \frac{y^{k*}_{b,2}}{(y^k_{b,1})^{\rho_b}\exp(2\alpha^k_b)} \Rightarrow \delta^m_2 \leq \delta^k_2 \ .$$

(Q.E.D)

Equity risk premium and Sovereign debt

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Abstract

A change in interest rate influences the cost of borrowing for the investors. This in turn affects the borrowing side of the economy. Domestic economy of the emerging markets and international sovereign debt market, such as those of Latin America and Asia, are affected by changes in the U.S. monetary policy. In 1994, the U.S. monetary policy was tightened that resulted in widening of sovereign spreads. In 1998, an opposite effect took place after loosening of the monetary policy. A rise in U.S. rates tends to increase the debt-service burden of the borrowing country, which reduces their ability to repay. We present a model, which uses a pricing kernel to evaluate risk and return in financial markets of a lending country while embedding it in a model of sovereign default of a borrowing country. Thus, we bring together two separate strands of literature and present a holistic model of international finance, which evaluates how pricing of risk affects the price of debt, spreads and behaviour of the borrowing country, i.e., its ability to borrow and smooth consumption.

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1 Introduction

A change in the U.S. monetary policy or the world market influences the country risk and cost of funds of developing countries like Latin America, Asia and East Europe. It affects the debtor country's ability to repay loans and to attract foreign investors. In 1994 when the U.S. monetary policy tightened, the country spreads widened in Latin America. In 1998, the reverse happened when the Fed eased its monetary policy; the sovereign spreads decreased (Figure 1, Arora and Cerisola (2001)). Data on Argentina show similar trend. The correlation between Fed fund rate and the sovereign spreads is around 60 percent for the period between 1983-2001 (Figure 2). In addition, some basic analysis illustrates that the U.S. Fed fund rate has a positive and significant influence on the magnitude of sovereign spread.

The key motivation of this paper is to study how choices and characteristics of a lending country influence the real business cycles and domestic economy of a borrowing country and the international sovereign debt market. Specifically, we focus our attention to the case of the U.S. and Argentina. As a source of funding, the prevailing world interest rate in the lending country influences not only the borrowing country's ability to borrow but also the default probability, level of debt and mean spreads in the international sovereign debt market. For instance, when times are bad, the borrowing country's government might find it difficult to roll over debt and seek new funding.

We introduce a model of international finance, which combines two separate economic frameworks into one. We follow closely and extend the work by Melino and Yang (2003), Eaton and Gersovitz (1981), Arellano (2008) and Lizarazo (2012). Melino and Yang (2003) incorporates properties, such as mildly pro-cyclical inter-temporal elasticity of substitution and strongly counter-cyclical risk aversion in the framework with Epstein-Zin preferences for lenders. This specification is very useful in the evaluation of risk and generating moments of asset returns in the financial market. Eaton and Gersovitz (1981) incorporate endogenous default risk in an incomplete market setup. Recent models, such as Arellano (2008) and Lizarazo (2012), on endogenous sovereign debt have managed to explain real business cycles statistics for the borrower. While these two frameworks have successfully generated the economic outcomes from the view of respective agents- borrower and lender, literature cries out for a model, which can combine these two sides together to yield a holistic model of international finance.

We develop a stochastic general equilibrium model of a small open economy which borrows from the rest of the world in an international lending and borrowing market. This paper extends the analysis of model on sovereign debt by introducing a risk averse lending country, which faces uncertainty of future income. The lending country's wealth is determined solely by its own income shocks and is not affected by the default decision of the borrowing country. The lending and borrowing in the international debt market takes place between governments of each country. They lend and borrow by issuance of bonds in the international market. As the international debt contracts are not enforceable, the borrowing government may choose to default. This results in an output penalty and exclusion from the international financial market. The domestic agents have access to the domestic financial markets, which are not integrated across countries. Thus, the households or the private agents of a country do not have access to the foreign financial market. We assume that the decisions made by agents in the small open economy do not influence fundamental variables (quantities) and prevailing prices in the rest of the world.

The pricing kernel of the lending country is derived from state dependent preferences and it is crucial in evaluation of risk. We use this pricing kernel in our baseline economy and match data on real business cycle statistics of the borrowing country, financial outcomes in the lending country and the international sovereign debt market. We find that when lending country receives a higher (lower) income shock, the contract on international lending charges a lower (higher) interest rate. In addition, we observe higher issuance of debt when the borrowing or the lending country receives a higher income shock. The paper also highlights the importance of lending country's characteristics by replacing a risk averse lending country with risk neutral. This specification fails on all fronts. The default probability and mean spread are very high as compared to the data and equity risk premium puzzle remains unsolved. We also look at the standard model of sovereign debt, which is unable to resolve equity risk premium puzzle, and thus the moments of asset return remain unaccounted for.

We find that the standard model with its pricing kernel has a property that cannot match high spreads without counter-factually high default probabilities. One explanation is that spread is completely tied to the default probability. In order to match the spread, we need a theory without generating high default probability. Since, the pricing of risk varies over cycles and the risk averse lending country demands a risk premium, the price of debt has to be very different from default probability. In standard models, variation of spread is explained by the default probability. However, in the baseline economy of this paper, the variation of spread is also driven by the pricing of risk, which are affected by the business cycles of the lending country. Thus, we obtain high spreads with low default probability. These findings are very striking given that we choose an i.i.d. instead of correlated income process for the lending country.

Literature review.

Arora and Cerisola (2001) studies how changes in the U.S. monetary policy – federal funds target rate – impact sovereign bond spreads in emerging economies like Latin America, Asia and East Europe. They present empirical evidences using secondary markets data[1] to show that spreads widen in emerging economies when U.S. monetary policy is tightened. In 1994 and then in 1998, a similar but opposite instance happened. The U.S. monetary policy tightened in 1994, which resulted in widened spreads. A rise in U.S. rates tends to increase the debt-service burden of the borrowing country, which reduces the ability to repay. Bernie et.al. (IMF, 2008) present empirical results in support of linkages between the developed and emerging financial markets. In general, emerging economies are influenced by the disturbances in the developed economies.

This paper marries two models together, wherein we will use more recent development in the field of sovereign debt and equity risk premium puzzle. The aim of this paper is to see how business cycle statistics of an emerging economy vary as we incorporate explicitly lending country's preferences. This paper enriches the standard business cycle model under endogenous default by assuming lenders to be risk averse and incorporating domestic and international financial markets.

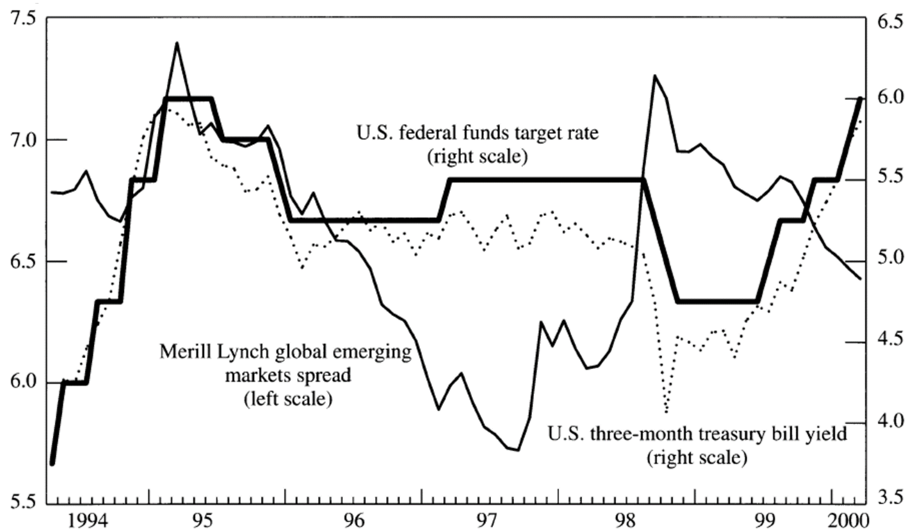
[1] Merrill Lynch Global emerging markets spread

In doing so, we calculate the financial and the real business cycle outcomes of a borrowing country in a stationary recursive equilibrium with default risk. We will also look at the (domestic) financial market outcomes of the lending country.

The economic literature has recently seen a lot of research in the field of emerging economies and its debt. These papers study the choice of level of debt and default decision by these economies. They focus on real business cycle models of international lending and borrowing under endogenous default decision. Most of the literature explores the effects of the borrowing country's characteristics on these decisions. The important factors could be output shocks, political stability, Hatchondo et al. (2009) etc. Some recent papers go beyond the borrower's characteristics to explore how lenders might affect the business cycles of these emerging economies, which tend to behave quite differently from that in the developed economies, Lizarazo (2012). In all these research papers, it has been highlighted that the emerging economies have business cycles that are more volatile, counter-cyclical rate of interest, which lags the cycles and the volatility of consumption is higher than the volatility of income, Neumeyer and Perri (2005).

Another strand of literature has attempted at explaining the equity risk premium puzzle and match asset returns and other financial variable with data. Equity risk premium is one of the most important financial figures used by investors for making investment decisions. A change in global interest rate might lead to a change in the riskiness of government bonds. This can further affect the returns on riskier assets, such as stocks, which should be reflected in change in the equity risk premium. Researchers have been trying to explain it and match it with data. This line of research has brought to light something important, which is called as equity risk premium puzzle. The economic models with standard preferences have failed to explain the large equity risk premium observed in data when the investors are risk averse. While equity risk premium puzzle does not exist when the lenders are highly risk averse, many explanations have come forth to illustrate shortcomings of these models with standard preferences. Many attempts have been made using models with habit preferences, Epstein-Zin preference etc. to account for mismatch between data and theory without much success.

Figure 1: Emerging Market Sovereign Spreads and U.S. Interest Rates

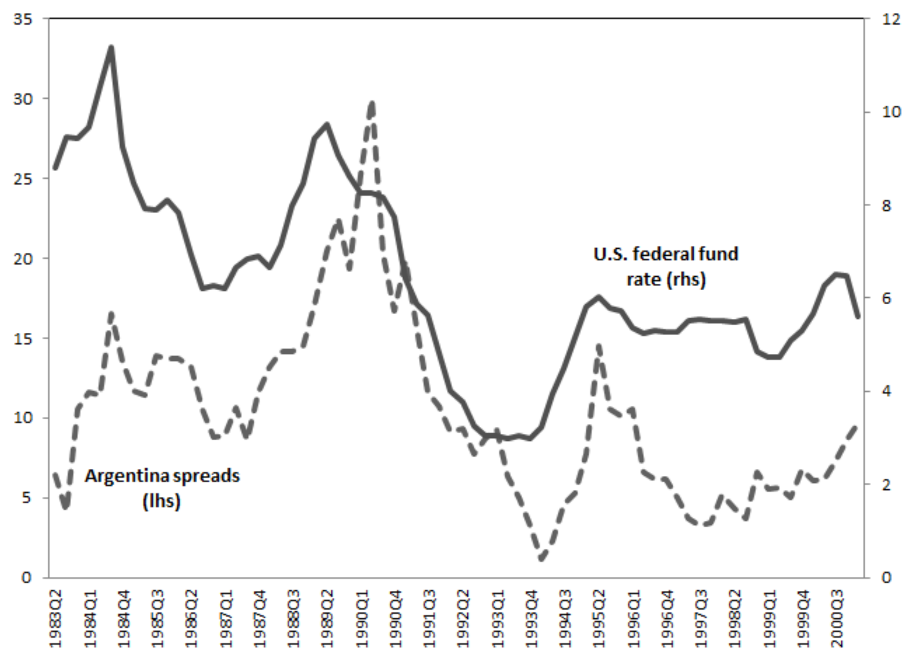


Source: Arora and Cerisola (2001)

Melino and Yang (2003) has highlighted that introducing state dependent preferences with Epstein-Zin formulation of recursive utility will help resolve the puzzle. They incorporate strongly counter-cyclical risk aversion and mildly pro-cyclical inter-temporal elasticity of substitution to the Epstein Zin preferences. The agents are sensitive to current state of economy. If a low state were realized today, agents would invest in assets which pays a higher return if low state is realized tomorrow and a low return if high state is realized tomorrow. However, if a high state were realized today then the agents would not be as sensitive as in the above case. They will invest in an asset, which pays similarly to either low or high state realization tomorrow.

Section 2 introduces the model and the environment with a discussion on agents and markets. It formally presents the agents' problems and equilibrium of this economy. Section 3 computes the model and discusses the quantitative analysis. We report results from calibration in this section.

Figure 2: Argentina Sovereign Spreads and U.S. Fed Fund Rate



Source: EMBI and Board of Governors of the Federal Reserve System

2 The Model

We study the world economy, which consists of a small open economy and a rest of the world. We follow the framework of Aguiar and Gopinath (2007) and Arellano (2008) to model default. We assume that the small open economy is small enough compared to the rest of the world so its policies do not affect international interest rates and consumption. It is the net borrower and rest of the world is the net lender in the international debt market.

Time is discrete and infinite, $t = \{0, 1, 2, ...\}$. The lending and the borrowing economies, $i \in \{l, b\}$, are inhabited by risk averse households of unit mass each and are ruled by benevolent government. The households are identical within each country and receive stochastic endowment stream every period. They have access to the domestic financial market through risky equity market and the risk-free bond market. The governments have access to the international debt market and trade a one-period risky bond. This debt contract is not enforceable and the borrowing government may choose to default on its debt obligation. The defaulting government is penalized with a fraction of its output immediately and it remains in financial autarky for a certain period.

Households. The households in the borrowing country have CRRA preferences over consumption, c_b, while the households in the lending country have Epstein-Zin preferences over consumption, c_l. The households of the borrowing and the lending country have access to the domestic capital market[2]. They can buy risky equity, s_i, at a price, q_i^e and risk free debt, a_i, at a price, q_i^{rf} . The equity pays stochastic dividend, $\theta_i(y_i)$, which is a function of the income realization y_i. Additionally, the households in the borrowing country receive a lump sum intra-period transfer of goods, $\mathcal{T}$, from its government.

The households in the borrowing country receives endowments y_b. It follows the AR(1) process,

$$log(y_b') = \rho_b log(y_b) + \epsilon_b', \tag{1}$$

where, the persistence parameter ρ_b is within the unit circle and the income shocks ϵ_b' are i.i.d and normally distributed $N(0, \varsigma_b^2)$. ς_b is the standard deviation of the income shocks

The households in the lending country receives income y_l and its growth rate, $\widetilde{g}_l'$, follows a Markov process such that

$$y_l' = \widetilde{g}_l' y_l, \tag{2}$$

where the Markov chain is ergodic and $\widetilde{g}_l'$ is positive.

The household in the borrowing country takes transfers $\mathcal{T}'$ as given. It is a stochastic process, which is endogenously determined from government's problem discussed later. It follows a Markov process and depends on the state of the economy, i.e. level of debt in the international debt market and income of the borrowing and the lending countries. The Bellman equation for the household in the borrowing country is given by

[2]Refer to the appendix for more details on the domestic market.

$$\mathcal{U}_b(w_b, y_b, \mathcal{T}) = \max_{\left\{s'_b \geq 0, a'_b \geq 0\right\}} \left\{u_b(c_b) + \beta_b E\left[\mathcal{U}_b(w'_b, y'_b, \mathcal{T}')\right]\right\}, \tag{3}$$

where $\beta_b \in (0, 1)$ is the borrower's discount factor and w_b is its wealth and y_b is the income realization. The utility function is strictly increasing and concave, i.e. $u'_b(.) > 0$ and $u''_b(.) < 0$ and satisfies the standard Inada conditions. The maximization is subjected to the budget constraint

$$c_b + q^e_b s'_b + q^{rf}_b a'_b = w_b \tag{4}$$

and the law of motion

$$w'_b = a'_b + s'_b \left[\theta'_b(y'_b) + q'^e_b\right] + \mathcal{T}' \tag{5}$$

The lending country is not affected by the borrowing country's characteristics as we assume a small open economy framework. The Bellman equation for the household in the lending country is given by

$$\mathcal{U}_l(w_l, y_l) = \max_{\left\{s'_l \geq 0, a'_l \geq 0\right\}} \left[c_l^{1-\varphi_l} + \beta_l \left(E\left[\mathcal{U}_l(w'_l, y'_l)^{1-\gamma_l}\right]\right)^{\frac{1-\varphi_l}{1-\gamma_l}}\right]^{\frac{1}{1-\varphi_l}} \tag{6}$$

where $\beta_l \in (0, 1)$ is the lender's discount factor, w_l is its wealth, y_l is its income realization, γ_l is its risk aversion and $\frac{1}{\varphi_l}$ is its inter-temporal elasticity of substitution. The utility function is strictly increasing and concave, i.e. $u'_b(.) > 0$ and $u''_b(.) < 0$ and satisfies the standard Inada conditions. The maximization is subjected to the budget constraint

$$c_l + q^e_l s'_l + q^{rf}_l a'_l = w_l \tag{7}$$

and the law of motion

$$w'_l = a'_l + s'_l \left[\theta'_l(y'_l) + q'^e_l\right] \tag{8}$$

We assume that the lending country is very big such that the default decision of the borrowing country does not affect its consumption. We also assume that $\beta_b < \beta_l$, so that small open economy b is the net borrower and rest of the world

l is the net lender.

Given a transfers function, $\mathcal{T}$, a recursive competitive equilibrium for an individual country i is given by a value function $\mathcal{U}_i$, decision rules g_i^s and g_i^a and the pricing functions q_i^e and q_i^{rf} such that,

(i) Given the pricing functions q_i^e and q_i^{rf}, the value function $\mathcal{U}_i$ and the decision rules g_i^s and g_i^a and solve the household's problem.

(ii) Markets clear

$$s_i' = g_i^s = 1$$

$$a_i' = g_i^a = 0$$

(iii) The equilibrium assets' price functions q_i^e and q_i^{rf} are consistent with the households optimization.

Government in the borrowing country. The government is benevolent and maximizes the utility of its representative households. It smooths consumption by trading a non-contingent bond, which pays a time- and state-invariant return. It receives qd' units of goods today and delivers d'units of goods tomorrow in the international debt market. For each state of the economy (y_b, y_l, d), the relationship between transfers $\mathcal{T}(y_b, y_l, d)$ and d' is determined by

$$\mathcal{T} = qd' - d \tag{9}$$

The state variables for the borrowing government are income shocks, $\mathbf{y}_b$ and $\mathbf{y}_l$, and the debt holding of the risky bond, $\mathbf{d}$; and the state variable for the lending government is $\mathbf{y}_l$. The aggregate income space is $\mathbf{y} = (\mathbf{y}_b, \mathbf{y}_l)$, the aggregate state space for the borrowing country is $(\mathbf{y}, \mathbf{d}) = (\mathbf{y}_b, \mathbf{y}_l, \mathbf{d}) = \mathcal{S}_b$ and the aggregate state space for the lending country is $\mathbf{y}_l = \mathcal{S}_l$. Let $\Pi(\mathbf{y}', \mathbf{y})$ represent the income transition matrix.

We denote the value function of the borrowing government as $V_b(\mathbf{y}, \mathbf{d})$. Let $V_b^d(\mathbf{y})$ denote the value function after default and $V_b^{nd}(\mathbf{y}, \mathbf{d})$ be the value function after repayment. The recursive formulation of the borrowing government's problem is expressed as

$$V_b(\mathbf{y},\mathbf{d}) = \max\{V_b^{nd}(\mathbf{y},\mathbf{d}), V_b^d(\mathbf{y})\}$$

When it chooses to default, it temporarily exits the international credit market and redeems access with probability λ. The decision to default leads to loss of output, y_b^d. The value function is,

$$V_b^d(\mathbf{y}) = u_b(y_b^d) + \beta_b \sum_{y_b'} \sum_{y_l'} \left[(1-\lambda)V_b^d(\mathbf{y}') + \lambda V^{nd}(\mathbf{y}', \mathbf{d}' = 0)\right] \Pi(\mathbf{y}', \mathbf{y})$$

If it chooses to repay the debt then the value function is,

$$V_b^{nd}(\mathbf{y},\mathbf{d}) = \max_{d'} \left\{ u(y_b - d + q.d') + \beta_b \sum_{y_b'} \sum_{y_l'} V_b(\mathbf{y}',\mathbf{d}')\Pi(\mathbf{y}',\mathbf{y}) \right\}$$

The borrowing government faces the resource constraint

$$c_b = y_b + \mathcal{T}$$

At the start of the period, the borrowing government takes the default decision after observing the income shocks. If it decides to repay its debt, then the borrowing government chooses d' in order to maximize the utility subject to the resource constraint. It takes the bond price schedule $q(\mathbf{y},\mathbf{d}')$ as given. The households in two countries choose the quantity of equity and risk-free assets taking prices as given. All agents consume.

The default set of the borrowing government is characterized by the aggregate income space $\mathbf{y}$ for which it finds optimal to default for a given level of debt $\mathbf{d}$. More precisely, $\mathbf{D}(\mathbf{y}',\mathbf{d}')$ is given by:

$$\mathbf{D}(\mathbf{y}',\mathbf{d}') = \{\mathbf{y}' \in \mathbf{Y} : V^{nd}(\mathbf{y}',\mathbf{d}') < V^d(\mathbf{y})\} \tag{10}$$

The repayment set is defined as the $\mathbf{A}(\mathbf{y}',\mathbf{d}')$

$$\mathbf{A}(\mathbf{y}',\mathbf{d}') = \{\mathbf{y}' \in \mathbf{Y} : V^{nd}(\mathbf{y}',\mathbf{d}') \geq V^d(\mathbf{y})\} \tag{11}$$

The value function of the lending government is $V_l(\mathbf{s})$ and is given by

$$V_l(\mathbf{y}_l) = \max_{c_l}\left\{u(c_l) + \beta_l \sum_{y_b'}\sum_{y_l'} V_l(\mathbf{y}_l')\Pi(\mathbf{y}',\mathbf{y})\right\}$$

The lending government faces resource constraint given by

$$c_l = y_l$$

Stationary Recursive Competitive Equilibrium. A stationary recursive competitive equilibrium for a world economy, which faces default risk in international debt market, is a set of value functions $V_b : \mathcal{S}_b \mapsto R$ and $V_l : \mathcal{S}_l \mapsto R$; policy functions for the borrowing household $g_b^s : \mathcal{S}_b \mapsto R$, $g_b^a : \mathcal{S}_b \mapsto R$ and $g_b^c : \mathcal{S}_b \mapsto R$; policy function for the borrowing government $g_b^d : \mathcal{S}_b \mapsto R$ and transfers function $g^T : \mathcal{S}_b \mapsto R$; policies for the lending households $g_l^s : \mathcal{S}_l \mapsto R$, $g_l^a : \mathcal{S}_l \mapsto R$ and $g_l^c : \mathcal{S}_l \mapsto R$; pricing functions q_l^e,q_l^{rf}, q_b^e,q_b^{rf} and q; a stationary measure π^* such that

(i) Given the pricing functions q_l^e and q_l^{rf}, the policy functions g_s^l, g_a^l and g_c^l solve the household's problem in the lending country and

(ii) Given the pricing functions q_b^e and q_b^{rf}, the transfers function g^T, the policy functions g_s^b, g_a^b and g_c^b solve the household's problem in the borrowing country and

(iii) Taking as given the borrowing government policy, g_d^b, households consumption g_c^b satisfies the resource constraint.

(iv) Taking as given the risky bond price function $q(\mathbf{y},\mathbf{d}')$, the governments policy functions g_d^b, the default set $\mathbf{D}(\mathbf{y}',\mathbf{d}')$ and the repayment set $\mathbf{A}(\mathbf{y}',\mathbf{d}')$ satisfy the borrowing government's optimization problem.

(v) Bonds prices $q(\mathbf{y},\mathbf{d}')$ incorporate the borrowing government's default probabilities and are consistent with lending government's optimization.

(vi) The domestic goods, equity and risk free bond market and the international risky bond markets clear

$$s_b' = g_b^s = 1 \text{ and } s_l' = g_l^s = 1$$

$$a_b' = g_b^a = 0 \text{ and } a_l' = g_l^a = 0$$

(vii) The invariant probability measure is consistent with the stochastic income process.

(viii) The equilibrium debt price function $q(\mathbf{y}, \mathbf{d}')$, asset price functions $q_b^e(\mathbf{y}, \mathbf{d}')$, $q_b^{rf}(\mathbf{y}, \mathbf{d}')$, $q_l^e(\mathbf{y}_l)$ and $q^{rf}(\mathbf{y}_l)$ are consistent with the optimization problems of the borrowing country's households, lending country's households and the borrowing government. The equilibrium in the international debt market implies that the price of the risky bond for state $(\mathbf{y}, \mathbf{d})$ is,

$$q(\mathbf{y}, \mathbf{d}') = \sum \sum_{(y_b', y_l') \in \mathbf{A}'} \Pi(\mathbf{y}', \mathbf{y}) \mathcal{M}_l(\mathbf{y}_l', \mathbf{y}_l) \tag{12}$$

where, $\mathcal{M}_l(\mathbf{y}_l', \mathbf{y}_l) = \beta_l \left(\frac{c_l'}{c_l}\right)^{-\varphi_l} \left(\frac{V_l(y_l')}{\mathcal{R}\big(V_l(y_l')\big)}\right)^{\varphi_l - \gamma_l}$, is the stochastic discount factor of the lending household and $\mathcal{R}(.)$ is a function defining the certainty equivalence of tomorrow's utility.

In the domestic market of the borrowing country, the price of equity

$$q_b^e(\mathbf{y}_b) = \sum_{y_b'} \Pi(\mathbf{y}_b', \mathbf{y}_b)(q_b^e(\mathbf{y}_b') + \mathbf{y}_b') \mathcal{M}_b(\mathbf{y}_b', \mathbf{y}_b)$$

where, $\mathcal{M}_b(\mathbf{y}_b', \mathbf{y}_b) = \beta_b \left(\frac{c_b'}{c_b}\right)^{-\gamma_b}$, is the stochastic discount factor of the borrowing household. The price of risk free debt, q_b^{rf}, in the domestic market is

$$q_b^{rf}(\mathbf{y}_b) = \sum_{y_b'} \Pi(\mathbf{y}_b', \mathbf{y}_b) \mathcal{M}_b(\mathbf{y}_b', \mathbf{y}_b)$$

In the lending country, the price of equity

$$q_l^e(\mathbf{y}_l) = \sum_{y_l'} \Pi(\mathbf{y}_l', \mathbf{y}_l)(q^e(\mathbf{y}_l') + \mathbf{y}_l')\mathcal{M}_l(\mathbf{y}_l', \mathbf{y}_l)$$

where, $\mathcal{M}_l(.)$ is the marginal rate of substitution.
The price of risk free debt

$$q_l^{rf}(\mathbf{y}_l) = \sum_{y_l'} \Pi(\mathbf{y}_l', \mathbf{y}_l)\mathcal{M}_l(\mathbf{y}_l', \mathbf{y}_l)$$

Equity Premium and Spread. We follow the standard approach of Mehra and Prescott (1985) to quantify the equity premium in the lending country. The equity premium, EQ_l, is given as the difference between un-conditional return on equity, R_l^e, and un-conditional return on risk-free debt, R_l^{rf}.

$$EQ_l = R_l^e - R_l^{rf} \tag{13}$$

Similarly, we quantify the financial variables for the borrowing country.

$$EQ_b = R_b^e - R_b^{rf} \tag{14}$$

The equity risk premium in the borrowing country, EQ_b, corresponds to the difference between un-conditional return on equity, R_b^e, and un-conditional return on risk-free debt of the borrowing country.

The spread,S, is the difference between un-conditional return on sovereign bond, R^d and un-conditional return on risk-free debt, R_l^{rf}. All details on un-conditional returns are provided in the Appendix.

3 Quantitative Analysis

3.1 Calibration and functional forms

We solve the model numerically and use quantitative results to discuss default, financial features (for e.g. equity risk premium) and business cycle properties (for e.g. interest rates spreads, output, and consumption) of this economy. We use parameters derived from the literature, Arellano (2008), for Argentina and from combined sources, data and Mehra and Prescott (1985), for the U.S. economy. They represent the borrowing and the lending countries in the model of this paper. A period in the model refers to a quarter in data, which represents time-period from 1983 : $Q3$ until 2001 : $Q3$.

Functional forms and parameters We use parameters from Arellano (2008), Mehra and Prescott (1985) and Melino and Yang (2003) for the borrowing and the lending country. We vary these parameters in order to answer how characteristics of the borrowing and the lending countries change the model outcomes with respect to data. To begin with, we adopt a specific baseline for the analysis. In this setting, the lending country has state dependent recursive preferences while the borrowing country has CES preferences, as considered in standard literature. The lending country is risk averse when low-income state is realized and it is risk neutral when high-income state is realized. The borrowing country is very impatient and risk averse.

Table 1 lists all those exogenous benchmark parameters, which are taken from Arellano (2008), Mehra and Prescott (1985), Melino and Yang (2003) and derived from data. The risk free world interest rate is 0.8 percent, which matches the quarterly yield on the five-year US Treasury bond. The probability of re-entry is 0.282, which matches 1.75 trade balance volatility[3]. Moreover, the output cost after default is set at 0.969 of the average output, which captures 5.53 percent debt service to GDP. Similar to Arellano (2008), we assume that default results in direct output cost, $\widetilde{y}_b$, of the following form

$$f(y_b) = \begin{cases} \widetilde{y}_b & \text{if } y_b > \widetilde{y}_b \\ \\ y_b & \text{otherwise.} \end{cases}$$

For the borrowing country, we set the CRRA, $\gamma_b = 2$, and the discount factor

[3]As in Aguiar and Gopinath (2007), probability of redemption implies that the economy cannot access international credit market for 2.5 years on average. Gelos et. al. (2003) reports it to be around three years in data.

at $\beta_b = 0.882$. The output of the borrowing country follows an AR(1) process with persistence and standard deviation set at $\rho_b = 0.945$ and $\varsigma_b = 0.025$, respectively. In case of lending country, we use the environment and parameters for the endowment process as presented in Mehra and Prescott (1985). The consumption growth is calculated so that the average growth rate of per capita consumption is 0.018, its standard deviation is 0.036. The first order serial correlation is -0.14, which translates into probability 0.43 of remaining in same state. We incorporate this parameter space and the stochastic discount factor from Melino and Yang (2003) in order to account for financial variables, as observed for the lending country's actual data. Melino and Yang (2003) target the estimates of historical average return on equity and risk free rate, which are at 7 percent and 0.8 percent, respectively. The standard deviation of equity and risk free rate is 0.165 and 0.056, respectively. With these targets in mind Melino and Yang (2003) estimate the rates on equity and risk free processes, i.e. values attained for different income realizations. These processes are then linked to the stochastic discount factor and the first order conditions with respect to equity and risk free bond. The first order conditions are derived from maximization problem of the agents subject to wealth accumulation constraint. They find specific parameter spaces of discount factors, risk aversion parameters and inter-temporal elasticity of substitution parameters, which satisfies the first order conditions[4] that match the equity and risk free rate processes exactly. One such parameter space is used for this paper. We set the lending country's discount factor at 0.98, the risk aversion parameter during recession is 23.25 and during boom it is 0.21, the intertemporal elasticity of substitution during recession is 2.1 and during boom, it is 2.98.

Discretization of shocks. We discretize the state space for the variables as follows. There are 21 grids for borrower's income y_b, 2 grids for lender's consumption (income) growth rate g_l' and 400 grids for debt. This particular choice of grids are considered as we want to compare, later, the results using the benchmark parameters and the parameters from Arellano (2008), Mehra and Prescott (1985) and Melino and Yang (2003). Accordingly, the income space is approximated using the procedure from Tauchen and Hussey (1991).

[4]Refer to the Appendix

Parameter	Description	Value
δ	Loss of output in autarky	0.969 E$[y_b]$
λ	Probability of re-entry	0.282
r^*	World interest rate	0.8%
Borrower		
β_b	Discount factor	0.882
γ_b	Risk aversion	2
ς_b	Standard deviation	0.025
ρ_b	Persistence Transitory shock	0.945
Lender		
β_l	Discount factor	0.98
γ_l	Risk aversion	$\{0.21, 23.25\}$
$\frac{1}{\varphi_l}$	Inter-temporal elasticity of substitution	$\{2.98, 2.1\}$
ς_l	Mean consumption growth	0.018
ρ_l	Variance consumption growth	0.0012
π	Probability of remaining in same state	0.43

Table 1: Baseline Parameter values

3.2 Calibration results

In this section, we report actual and simulated quarterly business cycles for Argentina from 1983 until 2001. The quantitative predictions are obtained by simulating the model over time to obtain short run and long run statistics. For short (or medium) run statistics, we consider 100 defaults events and 74 observations before the default. We report interest rate spread, output and consumption around default episodes. We also look at the volatility and correlation between these variables for the borrowing country. Then, we calculate the long-run average for interest rate spread, debt to output share and default probability. In addition, we measure financial variables such as return on equity, return on risk free bond, return on sovereign bond and equity risk premium.

Table 2 reports results on business cycle statistics and financial variables for actual data in column 1 and the baseline economy in column 2. In order to isolate the role of risk aversion, we also simulate the baseline under the assumption of a risk neutral lending country while keeping rest of the parameters fixed. The results from this exercise are reported in column 3. In the last experiment, results in column 4, we recalibrate the risk neutral lenders and match the default probability of the baseline economy.

In the baseline economy, we build on Melino and Yang (2003) using a pricing kernel, which helps resolve the equity risk premium puzzle and risk free rate puzzle, for the lending country. The pricing kernel provides a good theory about evaluation of risk in the lending country and international debt market as we embed it in the model of sovereign debt and default. A resolution of risk free rate puzzle ensures that the risk free rate in the lending country is low, as observed in data. Low risk free rate enables lenders to access funds at low cost of borrowing and invest in assets with higher return. The obvious choices are going to either domestic equity market or international debt market, which pay a higher risk premium.

The baseline economy focuses on specific structure - the borrowing and the lending countries are risk averse and the borrowing country is very impatient. The lending country is highly risk averse when the state of the economy is low while it is risk neutral when the state of the economy is high. The risk aversion, which is strongly counter cyclical, is set at 23.25 in low state and 0.21 in high state. The inter temporal elasticity of substitution is mildly pro-cyclical but highly responsive. It is set at 2.1 in low state and 2.98 in high state. The discount factor for the lending country is set at 0.98 and for the borrowing country at 0.882.

The current state of the economy in lending country affects the investment decision. As future is also important, a low-income state in the lending country today would encourage investors to invest in an asset, which promises a higher payment when a low-income state realizes tomorrow. This asset could pay relatively lower amount in higher income realization tomorrow. The lending country demands compensation during recession, as they are more risk averse. A high-income state today, which punctuates risk neutrality, does not highlight differences of such a magnitude in future particularly.

These features - strongly counter cyclical risk aversion and mildly pro-cyclical inter temporal elasticity of substitution -generate a pricing kernel, which is more accurate in evaluation of risk and return. The return on equity, 7.01 percent, is comparatively much higher than the risk free return, 0.80 percent, in the lending country. At these levels, the model matches the data very well with respect to equity risk premium and risk free rate (puzzles). The equity risk premium is around 6.21 percent and the risk free rate is low. The return on risky sovereign bond, 15.23 percent, issued by the borrowing country is higher than the return from equity market in the lending country. Here, we observe a low default rate, 4.43 percent, and a moderately high debt issuance as a share of output, 7.67 percent. These two effects, together, result in a high long-run mean spread, 14.43 percent, which is closer to the data. In medium (or short) run, around default episodes, the model over estimates the mean spread, 53.31 percent, as compared to 28.6 percent in data.

When the lender is risk neutral with CES preferences and the borrower is very impatient, with the discount factor set at 0.882, we are in a very different economy. The return on borrowing country's sovereign bond, 2.74 percent, is higher than the return on equity, 2.04 percent, in the lending country. Moreover, return on equity is same as return on risk free bond in lending country. Thus, the equity risk premium remains unresolved in this economy. These returns in various markets, international and domestic, leads to high debt issuance as a share of output, 16.49 percent, high default rate, 16.94 percent and thus a very high long run mean spread, 16.43. The medium run mean spread, 37.22 percent, is an overestimate of the data. This variation of the economy misses the data on several points for both, the borrowing and the lending country.

We re-calibrate the above economy, which has a risk neutral lending country and a patient borrowing country. The borrowing country has a discount factor set at 0.953. In this economy, the asset markets have very similar returns. The return on equity and risk free return in the lending country is 2.04 percent and the return on borrowing country's sovereign bond is 2.06 percent. The differences in these returns highlight moderate debt issuance as a share of output, 5.96 percent, and default rate, 2.99 percent. These two opposing effects lead to a low long run mean spread, 3.52 percent, which does not match data. Moreover, the returns on different assets do not resolve the equity risk premium puzzle. This model economy underestimates the medium (short) run mean spread, 23.63 percent, which is result contrary to the one in baseline model.

The model presented in this paper highlights the role of risk averse lending country. In three experiments, we simulate the economy to see how risk aversion alone changes results. Under the baseline economy, a low default probability at 4.43 is associated with high spread at 14.43 over the business cycle. However, in column 3 and column 4, the default probability and spread move one-to-one. When the preferences are changed from the baseline to a risk neutral lending country with everything else same, we get a high spread at 16.43 because default probability is high at 16.94. We observe this behaviour as the pricing of risk is incorrect. The pricing kernel does not react much to the level of debt. The borrowing country borrows a lot and tend to default a lot. When we recalibrate the model with risk neutral lending country to match the default probability we find that the spread is low at 3.52. This shows that the standard theory is missing an important link between spread and default probability. The relationship between spread and default probability is a necessary ingredient in determining the price of sovereign bonds.

All three model economies do well with respect to other business cycle statistics of the borrowing country. They are in line with empirical findings on emerging economies. The volatility of consumption is higher than the volatility of income, consumption is pro-cyclical and spread is counter cyclical. However, the equity risk premium, in the borrowing country's domestic financial market, is low. It stands at around 0.20 percent. It is worth reminding readers that the low equity risk premium could be attributed to the preference of the agent in the borrowing country. Our best guess is that if CES preference was replaced by the state dependent Epstein-Zin preference, we will be able to get a higher equity risk premium. The equity risk premium for emerging economies is quite difficult to estimate empirically as the actual data is available for short periods and it is highly volatile. Damodaran (2016) reports the annual equity risk premium of Argentina at 9.16 percent. He estimates Argentina's annual equity risk premium for 2010-12 using the method of relative equity market standard deviations under the assumption that the equity risk premium for the U.S. is 6 percent. This method uses annualized standard deviations of the S&P 500 and the MERVAL index in the U.S and Argentina. The key underlying assumption of this method is a linear relationship between equity risk premiums and equity market standard deviations. Salomons and Grootveld (2003) reports the average monthly equity risk premium for Argentina from 1976 until 2001 at 3.16 percent. Other studies, which estimate equity risk premium for emerging economies, and specifically for Argentina, find it to be higher than the developed countries.

Figure 3 plots the bond price schedules faced by the borrowing country with different preferences of the lending country. The bond price schedules in panel (a) and panel (b) corresponds to the borrowing country's income shocks 10 percent below and above the trend. The bond price schedules display standard properties across lending country's preferences, i.e. it is an increasing function of assets or higher level of debt is associated with lower bond price (or, higher interest rates). The figure displays the baseline economy under two scenarios: high and low-income realizations for the lending country (risk averse (high) and risk averse (low)). When the borrowing country receives a high-income shock, it can choose from a set of contract, which offers lower interest rates, as compared to when it receives a low-income shock. If the lending country also receives a high-income shock, and thus almost risk neutral, contracts with lower interest rates and higher-level debts are available in the international debt market. If, however, it receives a lower-income shock, higher interest rate is charged for the same level of debt. In case of a risk neutral lending country, a lower interest rate is offered, when compared with the state dependent (with high-income realization today) lending country, at lower levels of debt. This finding is in line with the description of the state dependent preferences of the lending country, which chooses different assets depending on current income realization.

Figure 4 presents the saving function of the borrowing country under different characteristics of the economic agents. The level of debt, d', is higher when the borrowing country is in boom[5]. Inn panel (a), high-income state of the borrowing country, under persistent income, ensures repayment of debt. On the contrary, recessions in the borrowing country limits the capacity to borrow in the international debt market. A high level of debt, d', is issued for high level of initial debt, d, when the lending country is risk neutral and the borrowing country is very impatient. The debt level, d', is only slightly lower when the lending country has counter-cyclical risk aversion and pro-cyclical inter temporal elasticity of substi-

[5]Higher debt corresponds to a high negative value on the x-axis and the y-axis of Figure 4.

tution. Risk aversion has one of the direct dampening impact on debt issuance. It is noteworthy that debt issuance, d', is lower during recession than during boom in the lending country. Moreover, the re-calibrated economy with risk neutral lending country allows lower debt issuance, d', when the initial debt level, d, is already very high, as compared to the other preference specifications. In panel (b), the borrowing country's income is below the trend. Here, the borrowing and lending does not take place at higher levels of initial debt. However, when the initial debt level is not so high, the debt issuance depends on the lending country's preferences. Higher level of debt is issued with state dependent preferences during boom in the lending country. Similar level of debt is observed in case of a risk neutral lending country. However, in case of re-calibrated economy with risk neutral lending country and the economy with state dependent preferences during recession in the lending country, the debt issuance is much lower.

The next step involves further analysis by using the model in this paper as the starting point. An extension would involve looking at the returns on equity, returns on bond and equity risk premium in the borrowing country's domestic financial markets when default episodes occur. These are expected to be much different from the long run average as default affects the output of the borrowing country, though that of the lender remains unchanged in this model. Currently, it is difficult to analyse complete (full) influence of lender's characteristics on borrowing country's decisions due to huge size of the modelling data (matrices) and limited states for lending country's income. One possibility is to extend the analysis from an i.i.d. income process to a Markov income process and allow for some correlation between the incomes of the borrowing and the lending countries.

4 Conclusion

The real and financial markets of small open economies are vulnerable to changes in interest rates of the large open economies. Periods like in 1994 and 1998 had been a very clear indication that the Latin American, as well as Asian, economies were affected by changes in the monetary policy of the U.S. and hence its interest rates. We present a model, which uses a pricing kernel to evaluate risk (and return) in financial markets while embedding it in a model of sovereign default.

In doing do, we bring together, two separate strands of literature, and present a holistic model of international finance.

The first strand sets its attention on the financial markets of the developed countries, such as the U.S. The pricing kernel, which we derive from state dependent preferences of the lending country, enables us to resolve the puzzle associated with equity risk premium. Melino and Yang (2003) emphasizes the role of (moderately) pro-cyclical inter temporal elasticity of substitution and (strongly) counter-cyclical risk aversion in resolving it. These preferences affect the choices that lending country makes while investing in assets, such as domestic equity, international sovereign bond etc. Even though, portfolio allocation is not the focal point of this paper, we show that the investment decisions of investors depend on various aspects, such as income shocks in two countries, characteristics of the borrowing and the lending country etc.

In order to align with the aim of this paper we look at how lending country might influence the sovereign debt market and the domestic economy of the borrowing country. This brings us to the second strand of literature, which focuses on the endogenous default model for emerging economies, such as Argentina. These models have been successful, largely, in explaining business cycle statistics of emerging economies when contracts in the international lending and borrowing market are unenforceable.

We find that the model presented in this paper, after incorporating some key features, not only matches data concerning the real business cycle statistics of the borrowing country but also assesses the risk very well for the lending country. We observe that state dependent preferences are key in evaluating risk more accurately. We also find that when lending country receives a higher (lower) income shock, the contract on international lending charges a lower (higher) interest rate. In addition, we observe higher issuance of debt when the borrowing or the lending country receive a higher income shock.

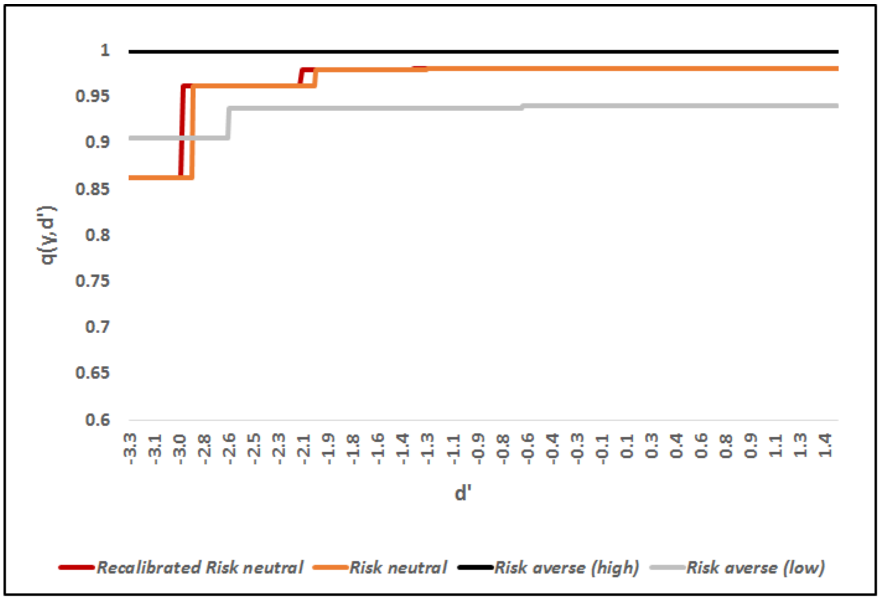
Risk neutral preferences, under similar conditions, fail to explain the observed statistics for both- the borrowing and the lending country. The default probability, mean debt to output ratio and mean spread are very high as compared to the data. Moreover, the puzzle on equity risk premium remains unsolved. Re-calibrating the model with risk neutral lending country by incorporating a patient borrower, as in the standard literature, explain the business cycle statistics for the borrowing country. However, re-calibrating the parameters does not allow the model to explain the asset markets in the lending country.

The three economies in this paper illustrate that the baseline economy's spread is very different from the default probability. The standard model can generate huge spreads only with a high default probability. We have established evaluation of risk, which changes with the business cycles of the lending country. The level of spread is different from the default probability due to existence of risk premium. Thus, an accurate pricing kernel is very important in accounting for risk in the international debt market.

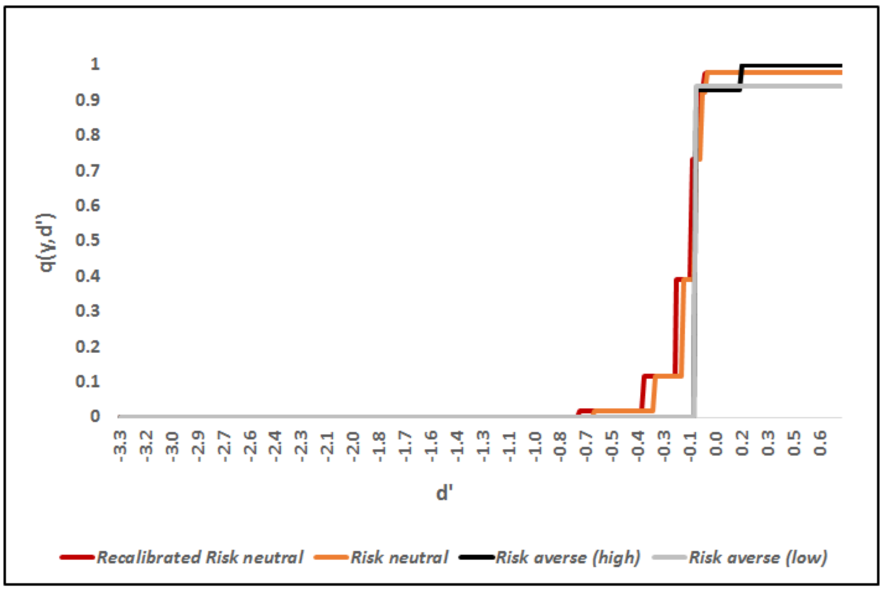
The model presented in this paper is the first attempt in bringing features of lending and the borrowing country explicitly, while accounting for statistics of each of the economies together. This paper has successfully highlighted the importance of lending country's characteristics in defining not only the international sovereign debt market but also the domestic economy of the borrowing country. Thus, the model in this paper is very useful as the starting point for any future analysis, which has an intention to investigate a research question pertaining to international finance, specifically to international debt market.

	Data	Risk averse Lender	Risk neutral Lender	Risk neutral Re calibrated
Parameters				
β_b	-	0.88	0.88	0.95
Default episodes				
Output (Y_b)	-14.21	-13.23	-13.50	-12.59
Consumption (C_b)	-16.01	-13.07	-13.23	-12.45
Rate of interest spread (S)	28.6	53.31	37.22	23.63
Standard dev				
$\sigma(Y)$	7.78	5.81	5.25	5.79
$\sigma(C)$	8.59	7.14	7.13	6.28
$\sigma(S)$	5.58	20.91	17.91	5.76
Correlations				
$\rho(C,Y)$	0.98	0.89	0.86	0.97
$\rho(S,Y)$	-0.88	-0.15	-0.69	-0.33
$\rho(S,C)$	-0.89	-0.28	-0.66	-0.40
Mean spread	10.25	14.43	16.43	3.52
Mean Debt/Output	-	7.67	16.49	5.96
Default probability	3	4.43	16.94	2.99
EQ_b	-	0.20	0.20	0.21
R^d	-	15.23	2.74	2.06
EQ_l	6	6.21	0.00	0.00
R_l^e	-	7.01	2.04	2.04
R_l^{rf}	-	0.80	2.04	2.04

Table 2: Data and Model

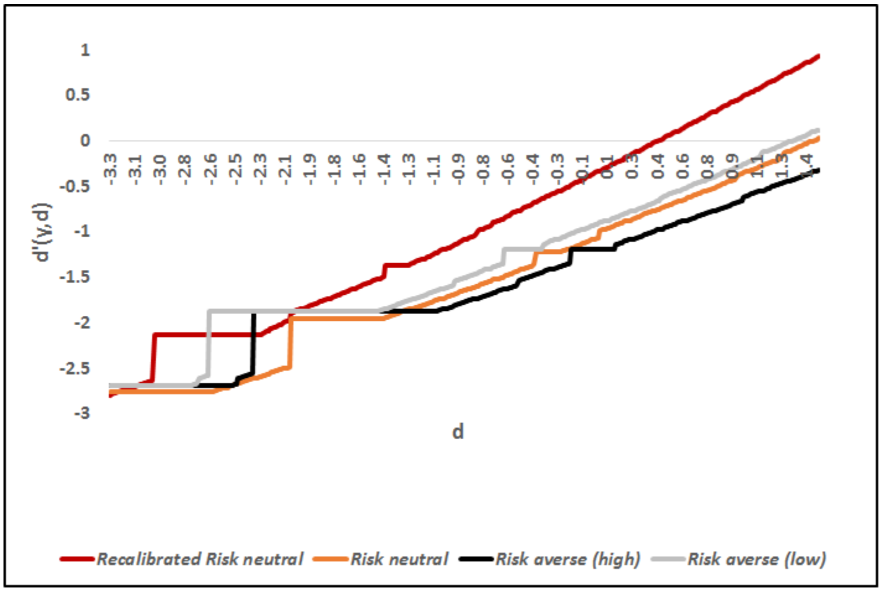


(a) High income realization for borrower

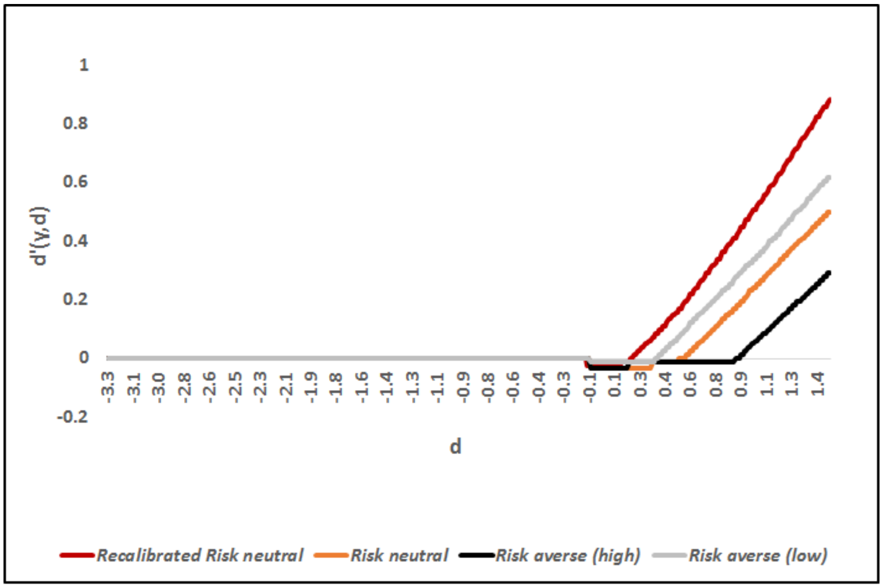


(b) Low income realization for borrower

Figure 3: Bond Price Schedule



(a) High income realization for borrower



(b) Low income realization for borrower

Figure 4: Saving function

5 Appendix

Household's problem in lending country with Epstein-Zin preferences. In the section we use time based subscripts in order to illustrate the importance of state dependent preferences for the lender (also, subscript 'l' is removed for simplicity). The household's problem with Epstein-Zin preferences in the lending country can be written as follows:

$$\mathcal{U}_t(w_t, y_t) = \max_{\{s_{t+1}\geq 0, a_{t+1}\geq 0\}} \left[c_t^{1-\varphi_t} + \beta_t \left(E\left[\mathcal{U}_t(w_{t+1})^{1-\gamma_t} \right] \right)^{\frac{1-\varphi_t}{1-\gamma_t}} \right]^{\frac{1}{1-\varphi_t}}$$

subjected to following constraints

$$c_t + q_t^e s_{t+1} + q_t^{rf} a_{t+1} = w_{t+1}$$

the law of motion of wealth, where $\theta(y_{t+1}) = y_{t+1}$

$$w_{t+1} = a_{t+1} + s_{t+1} \left[\theta(y_{t+1}) + q_{t+1}^e \right]$$

and the law of motion of income with $\widetilde{g}_{t+1}$ as growth rate

$$y_{t+1} = \widetilde{g}_{t+1} y_t$$

The first order conditions w.r.t s_{t+1} gives

$$q_t^e = E_t \left[\beta_t \frac{c_{t+1}^{-\varphi_{t+1}}}{c_t^{-\varphi_t}} \left(\mathcal{R}\left(\mathcal{U}_{t+1}\right)^{\gamma_t - \varphi_t} \mathcal{U}_{t+1}^{-\gamma_t + \varphi_{t+1}} \right) \left(q_{t+1}^e + y_{t+1} \right) \right] \tag{15}$$

where, $\mathcal{R}_t\left(\mathcal{U}_{t+1}\right) = \left[E_t \left(\mathcal{U}_{t+1}^{1-\gamma_t} \right) \right]^{\frac{1}{1-\gamma_t}}$. Assume that the price is homogenous of degree one then $q_t^e = q^e y_t$. Also, a recursive competitive equilibrium of goods market implies $\widetilde{g}_{t+1} = \frac{y_{t+1}}{y_t} = \frac{c_{t+1}}{c_t}$. Thus,

$$q_t^e = E_t \left[\beta_t \widetilde{g}_{t+1}^{1-\varphi_{t+1}} y_t^{\varphi_t - \varphi_{t+1}} \left(\mathcal{R}\left(\mathcal{U}_{t+1}\right)^{\gamma_t - \varphi_t} \mathcal{U}_{t+1}^{-\gamma_t + \varphi_{t+1}} \right) \left(q_{t+1}^e + 1 \right) \right] \tag{16}$$

In a **special case** of this general specification, if $\varphi_t = \varphi_{t+1}$ and $\gamma_t = \gamma_{t+1}$ then the above equation reduces to the standard price equation.

$$q_t^e = E_t\left[\beta \widetilde{g}_{t+1}^{-\varphi}\left(\frac{\mathcal{U}_{t+1}}{\mathcal{R}\left(\mathcal{U}_{t+1}\right)}\right)^{-\gamma+\varphi}\left(q_{t+1}^e+1\right)\right] \tag{17}$$

where, $\beta \widetilde{g}_{t+1}^{-\varphi}\left(\frac{\mathcal{U}_{t+1}}{\mathcal{R}(\mathcal{U}_{t+1})}\right)^{-\gamma+\varphi}$ is the '*stochastic*' marginal rate of substitution. Similarly, the first order conditions w.r.t a_{t+1} gives

$$q_t^{rf} = E_t\left[\beta_t \frac{c_{t+1}^{-\varphi_{t+1}}}{c_t^{-\varphi_t}}\left(\mathcal{R}\left(\mathcal{U}_{t+1}\right)^{\gamma_t-\varphi_t}\mathcal{U}_{t+1}^{-\gamma_t+\varphi_{t+1}}\right)\right] \tag{18}$$

In a **special case**, eq(17) is given by

$$q_t^{rf} = E_t\left[\beta \widetilde{g}_{t+1}^{-\varphi}\left(\frac{\mathcal{U}_{t+1}}{\mathcal{R}\left(\mathcal{U}_{t+1}\right)}\right)^{-\gamma+\varphi}\right] \tag{19}$$

Here on, we follow the notations used in the paper, i.e. drop time subscripts , t.

Domestic Capital Market of the lending country. The domestic capital markets comprises of equity and risk free debt. We have used a variation of the Mehra and Prescott (1985) to calculate the returns for each of these investments. In discreet case, the price of equity, $q_l^e(\mathbf{y}_l)$, is rewritten as

$$q_l^e(\mathbf{y}_l) = \sum_{y_l'} \Pi(\mathbf{y}_l', \mathbf{y}_l)(q_l^e(\mathbf{y}_l') + \mathbf{y}_l')\mathcal{M}_l(\mathbf{y}_l', \mathbf{y}_l)$$

where, $\mathcal{M}_l(.)$ is the marginal rate of substitution. The realized return on equity is given by

$$r_l^e(\mathbf{y}_l', \mathbf{y}_l) = \left[\frac{q_l^e(\mathbf{y}_l') + \mathbf{y}_l'}{q_l^e(\mathbf{y}_l)} - 1\right]$$

The equilibrium in the domestic market implies that the conditional return on the equity is $\widetilde{R}_l^e(\mathbf{y}_l)$ and the un-conditional return is R_l^e

$$\widetilde{R}_l^e(\mathbf{y}_l) = \sum_{y_l'} \Pi(\mathbf{y}_l', \mathbf{y}_l) r_l^e(\mathbf{y}_l\prime, \mathbf{y}_l) \tag{20}$$

$$R_l^e = \sum_{y_l} \widetilde{\Pi}(\mathbf{y}_l)\widetilde{R}_l^e(\mathbf{y}_l) \tag{21}$$

The price of risk free debt, $q_l^{rf}(\mathbf{y}_l)$, is

$$q_l^{rf}(\mathbf{y}_l) = \sum_{y_l'} \Pi(\mathbf{y}_l', \mathbf{y}_l)\mathcal{M}_l(\mathbf{y}_l', \mathbf{y}_l)$$

The equilibrium in the domestic risk-free bond market implies that the un-conditional return on the risk-free bond R_l^{rf} is given as

$$R_l^{rf} = \sum_{y_l} \widetilde{\Pi}(y_l) \left[\frac{1}{q_l^{rf}(\mathbf{y}_l)} - 1 \right] \tag{22}$$

Domestic Capital Market of the borrowing country. The domestic capital markets comprises of equity and risk free debt. We use a variation of the Mehra and Prescott (1985) to calculate the returns for each of these investments. The price of equity, $q_b^e(\mathbf{y})$, is

$$q_b^e(\mathbf{y}_b) = \sum_{y_b'} \Pi(\mathbf{y}_b', \mathbf{y}_b)(q_b^e(\mathbf{y}_b') + \mathbf{y}_b')\mathcal{M}_b(\mathbf{y}_b', \mathbf{y}_b)$$

where, $\mathcal{M}_b(.)$ is the marginal rate of substitution of the borrowing country's households. The return on equity for an income realization is given by

$$r_b^e(\mathbf{y}_b', \mathbf{y}_b) = \left[\frac{q_b^e(\mathbf{y}') + \mathbf{y}'}{q_b^e(\mathbf{y})} - 1 \right]$$

The equilibrium in the domestic market implies that the conditional return on the equity is $\widetilde{R}_b^e$ and the un-conditional return is R_b^e

$$\widetilde{R}_b^e(\mathbf{y}_b) = \sum_{y_b'} \Pi(\mathbf{y}_b', \mathbf{y}_b) r_b^e(\mathbf{y}_b', \mathbf{y}_b) \tag{23}$$

$$R_b^e = \sum_{y_b} \widetilde{\Pi}(\mathbf{y}_b)\widetilde{R}_b^e(\mathbf{y}_b) \tag{24}$$

The price of risk free debt, $q_b^{rf}(\mathbf{y})$, is

$$q_b^{rf}(\mathbf{y}_b) = \sum_{y_b'} \Pi(\mathbf{y}_b', \mathbf{y}_b)\mathcal{M}_b(\mathbf{y}_b', \mathbf{y}_b)$$

The equilibrium in the domestic risk-free bond market implies that the expected return on the risk-free bond R_b^{rf} is given as

$$R_b^{rf} = \sum_{y_b} \widetilde{\Pi}(\mathbf{y}_b) \left[\frac{1}{q_b^{rf}(\mathbf{y}_b)} - 1 \right] \tag{25}$$

The price of government debt, $q(\mathbf{y}, \mathbf{d})$, is

$$q(\mathbf{y}, \mathbf{d}) = \sum \sum_{(y_b', y_l') \in \mathbf{D}} \Pi(\mathbf{y}', \mathbf{y})\mathcal{M}_l(\mathbf{y}_l', \mathbf{y}_l) \tag{26}$$

The equilibrium in the domestic risk-free bond market implies that the expected return on the risk-free bond R^d is given as

$$R^d = \sum_{y_b} \sum_{y_l} \widetilde{\Pi}(\mathbf{y}) \left[\frac{1}{q(\mathbf{y}, \mathbf{d})} - 1 \right] \tag{27}$$

References

Aguiar, M. and G. Gopinath (2007). Emerging market business cycles: The cycle is the trend. *Journal of political Economy 115*(1), 69–102.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *The American Economic Review*, 690–712.

Arora, V. and M. Cerisola (2001). How does us monetary policy influence sovereign spreads in emerging markets? *IMF Economic Review 48*(3), 474–498.

Damodaran, A. (2016). Equity risk premiums (erp): Determinants, estimation and implications–the 2016 edition.

Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies 48*(2), 289–309.

Hatchondo, J. C., L. Martinez, and H. Sapriza (2009). Heterogeneous borrowers in quantitative models of sovereign default. *International Economic Review 50*(4), 1129–1151.

Lizarazo, S. V. (2012). Default risk and risk averse international investors. *Journal of International Economics*.

Mehra, R. and E. C. Prescott (1985). The equity premium: A puzzle. *Journal of monetary Economics 15*(2), 145–161.

Melino, A. and A. X. Yang (2003). State-dependent preferences can explain the equity premium puzzle. *Review of Economic Dynamics 6*(4), 806–830.

Neumeyer, P. A. and F. Perri (2005). Business cycles in emerging economies: the role of interest rates. *Journal of monetary Economics 52*(2), 345–380.

Salomons, R. and H. Grootveld (2003). The equity risk premium: emerging vs. developed markets. *Emerging markets review 4*(2), 121–144.

Tauchen, G. and R. Hussey (1991). Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. *Econometrica: Journal of the Econometric Society*, 371–396.